**MA 180 Lecture**
**Chapter 7**
**College Algebra and Calculus by Larson/Hodgkins**
**Limits and Derivatives**

## 7.4) Some Rules of Differentiation

In this section we will learn important rules that will help us arrive at the derivative of a function easily. We will use these rules over the limit definition of the derivative.

### The Constant Rule

The derivative of a constant function is zero. That is:

$\frac{d}{dx}[c] = 0$, where $c$ is a constant.

Explain why this is true using the algebraic definition and by interpreting the graph.

Examples:
Find

$\frac{d}{dx}[9] =$

$\frac{d}{dx}[\pi] =$

### The (Simple) Power Rule

$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$, $n$ is any real number

Examples:

$\frac{d}{dx}\left[x^7\right] = 7x^6$

$\frac{d}{dx}\left[\sqrt{x}\right] = \frac{d}{dx}\left[x^{1/2}\right] = \frac{1}{2}x^{-1/2}$

$\frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}\left[x^{-3}\right] = -3x^{-4}$

Note in the last two examples that we do not drop the derivative operator when we simplify. It gets dropped only after we find the derivative.

Find the following derivatives.

| | |
|---|---|
| $\frac{d}{dx}\left[x^{10}\right]=$ | $\frac{d}{dx}\left[x^{4}\right]=$ |
| $\frac{d}{dx}\left[\sqrt[3]{x}\right]=$ | $\frac{d}{dx}\left[\frac{1}{x^{5}}\right]=$ |

Remember the derivative is a *function* that tells you the slope of the tangent line for any *x* in the domain of the original function.

Example:

Find the slope of the tangent line to the graph of $f(x)=x^3$ when $x=-2,-1,0,2$. Consider the graph and interpret your results.

**The Constant Multiple Rule**

If *f* is a differentiable function of *x* and *c* is a real number, then $\frac{d}{dx}\left[cf(x)\right]=cf'(x)$, $c$ is a constant.

In other words, if a function is multiplied by a constant, you find the derivative of the function and multiply the constant (carry the constant.)

Verify this using the definition of the derivative.

Examples:

Find the derivatives.

| | |
|---|---|
| $\frac{d}{dx}\left[5x^{7}\right]=$ | $\frac{d}{dx}\left[\frac{x}{3}\right]=$ |
| $\frac{d}{dx}\left[10\sqrt{x}\right]=$ | $\frac{d}{dx}\left[\frac{-3}{x^{8}}\right]=$ |
| $\frac{d}{dx}\left[4\pi x\right]=$ | $\frac{d}{dx}\left[5\sqrt[2]{x^{3}}\right]=$ |

Exercise: Be mindful of parenthesis when differentiating.

Find the derivative of the following functions.

| |
|---|
| $y=\frac{3}{4x^{2}}$ |
| $y=\frac{3}{(4x)^{2}}$ |
| $y=\frac{3}{4x^{-2}}$ |
| $y=\frac{3}{(4x)^{-2}}$ |

The next rule involves the addition or subtraction of two functions. We find that the derivative of the sum is the sum of the derivatives.

**The Sum and Difference Rules**

The derivatives of the sum or difference of two differentiable functions is the sum or difference of their derivatives.

*Sum Rule* $$\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x)$$

*Difference Rule* $$\frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$$

We can verify this using the definition of the derivative.

Examples: Find the derivatives.

| |
|---|
| $\frac{d}{dx}\left[6x^3-5x+8\right]=$ |
| $\frac{d}{dx}\left[\frac{3x^4-2}{x}\right]=$ |

Find the slope of the graph of $f(x) = 5x^3 - 4x + 7$ at the point *(2,39)*.

Find the equation of the tangent line to the graph of $f(x) = 5x + \sqrt{x}$ at the point *(4,22)*.

Determine the point(s), if any, at which the graph of the function has a horizontal tangent line.

$y = -x^4 + 3x^2 - 1$