MA 180 Lecture
Chapter 5
College Algebra and Calculus by Larson/Hodgkins
Systems of Equations and Inequalities

5.1) Solving Systems Using Substitution
5.2) Solving Systems Using Elimination

Sometimes we have mathematical equations involving more than one variable that we wish to solve for, these are called **systems of equations**. If we have two variables to solve for then we will need to equations to solve for exact values of those variables. In fact, if we have *n* variables to solve for then we need *n* equations. Note that if you have fewer than *n* equations then either the system has no solution, or it has multiple solutions.

There are four different ways to solve a system of equations, but we will only focus on two methods in this class. The first is the method of substitution.

Method of Substitution
1. *Solve* one of the equations for one variable in terms of the other.
2. *Substitute* the expression found in Step 1 into the other equation to obtain an equation in one variable.
3. *Solve* the equation obtained in Step 2.
4. *Back-Substitute* the value found in Step 3 into the expression obtained in Step 1 to find the value of the other variable.
5. *Check* that the solution satisfies each of the original equations.

Example: Solve

$$\begin{cases} x + y = 6 \\ x - y = 4 \end{cases}$$

Solve:

$$\begin{cases} x^2 + 4x - y = 7 \\ 2x - y = -1 \end{cases}$$

Solve:

$$\begin{cases} 2x^2 - y = 1 \\ x + y = -2 \end{cases}$$

The Method of Elimination

In this second method the idea is to eliminate one of the variables in order to solve for the other. The way we do this is by obtaining, for one of the variables, coefficients that differ only in sign, so that when we add the two equations we eliminate this variable.

The Method of Elimination To use the **method of elimination** to solve a system of two linear equations in *x* and *y*, use the following steps.
1. Examine the system to determine which variable is easiest to eliminate.
2. Obtain coefficients of *x* (or *y*) that differ only in sign by multiplying all terms of one or both equation by suitably chosen constants.
3. Add the equations to eliminate one variable and solve the resulting equation.
4. Back-substitute the value obtained in Step 3 into either of the original equations and solve for the other variable.
5. Check your solution in both of the original equations.

Example: Solve

$$\begin{cases} 2x - 3y = 5 \\ 5x + 3y = 9 \end{cases}$$

Solve:

$$\begin{cases} -2x + 7y = -15 \\ 3x - 5y = 6 \end{cases}$$

$$\begin{cases} 3x - 5y = 2 \\ -6x + 10y = 9 \end{cases}$$

$$\begin{cases} 3x + 5y = 7 \\ 9x + 15y = 21 \end{cases}$$

Notes: I will sometimes refer to the “method of elimination” as the “method of combination.”

We can also find solutions by graphing the two equations and the point of intersection of the two graphs corresponds to the solution to the system of equations. When there is exactly one point of intersection we say the system is **independent (consistent).** When there are an infinite set of solutions we say the systems is **dependent (consistent).** However, if there are no solutions (the graphs do not intersect) then we say the solution is **inconsistent.**

For this course, it is not necessary to use on system over the other, so feel free to use the method you are more familiar with or that works best for that specific problem. Ignore directions in the homework directing you to use one method over the other.

Solve the following using either method.

$$\begin{cases} 2x + y = 120 \\ x + 2y = 120 \end{cases}$$

$$\begin{cases} xy = 3 \\ y = \sqrt{x-2} \end{cases}$$