MA 180 Lecture
Chapter 4
College Algebra and Calculus by Larson/Hodgkins
Exponential and Logarithmic Functions

4.2) Logarithmic Functions

The function $f(x)=a^x$ passes the horizontal line test and therefore has an inverse function. Since it is a transcendental function, we cannot define the inverse algebraically. Instead we use a new type of function called the logarithmic functions (or "log" functions.) The log function is actually defined to be the inverse of the exponential function. It undoes raising numbers to a power.

Definition

For $x>0$, $a>0$, and $a\neq 1$,then $y=\log_a x$ if and only if $x=a^y$.

The function given by $f(x)=\log_a x$ is called the **logarithmic function with base *a***.

We say $y=\log_a x$ is the logarithmic form of the same equation $x=a^y$ which is called the exponential form. We think of a logarithm as being an exponent. It answers the question, "a raised to what power is x?" Then *y* is the power.

Examples:

$\log_2 8=3$ since $8=2^3$

Find the following:

$\log_2 32=$	$\log_2\left(\frac{1}{4}\right)=$	$\log_2 1=$
$\log_3 9=$	$\log_9 3=$	$\log_4 64=$
$\log_{374859}(374859)=$	$\log_{398}1=$	$\log_{10}(0.10)=$

Since the log function is the inverse function of the exponential function, it follows that the domain of the log function is the range of an exponential function. We learned in the last section that the range of an exponential function is $(0,\infty)$. Therefore the domain of any log function is $(0,\infty)$. We call *x* the **argument** of the log function for $\log_a(x)$.

Evaluate $\log_{10}(500)$.

Since 500 is not an exact power of ten we cannot find an exact value for this log. It is considered simplified as written (and therefore an exact value.) However, if we needed a decimal approximation for this we would plug it into our calculator. There should be a "log" button on the calculator. After plugging it in, you should get about 2.7. (This answer makes sense since we know 10 to the power of 2 is 100 and to the power of 3 is 1000, so the answer is somewhere in between. Note it is not exactly half way.)

Properties of Logarithms
1. $\log_a 1 = 0$ because $a^0 = 1$
2. $\log_a a = 1$ because $a^1 = a$
3. $\log_a a^x = x$ and $a^{\log_a x} = x$ (Inverse Property)
4. If $\log_a x = \log_a y$, then $x = y$ (One-to-One Property)

Examples: Solve

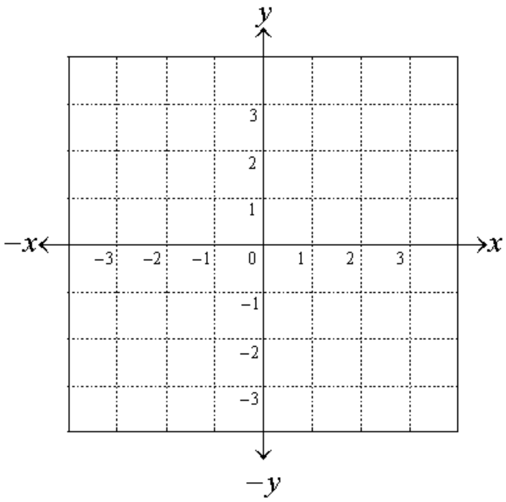
$$\log_2 x = \log_2 7$$

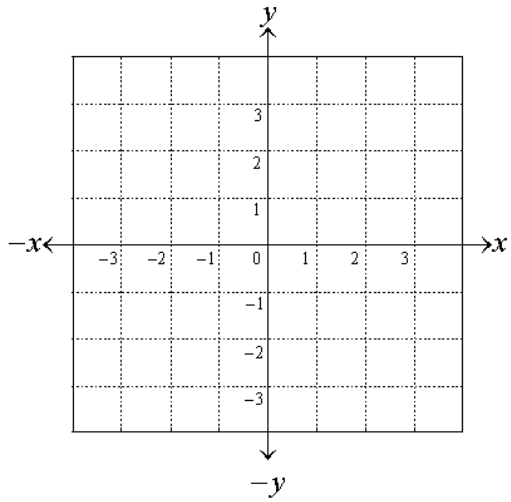
$$\log_4 x = 2$$

$$\log_6 6^{52} = x$$

We find the graph of a logarithmic function with base *a* by sketching the appropriate exponential function with base *a* and reflecting it across the line $y = x$.

Sketch both $f(x) = a^x$ and $g(x) = \log_a x$, *a>1*





Characteristics of Logarithmic Functions
Domain $(0, \infty)$
Range $(-\infty, \infty)$
Intercept $(1,0)$
Increasing
One-to-one, therefore has an inverse
y-axis is a vertical asymptote
Continuous
Reflection of graph of $f(x) = a^x$ about the line $y = x$

Definition

The function defined by $f(x) = \log_e(x) = \ln(x)$, $x > o$ is called the **natural logarithmic function.**

Properties of the Natural Logarithm
1. $\ln 1 = 0$ because $e^0 = 1$
2. $\ln e = 1$ because $e^1 = e$
3. $\ln e^x = x$ and $e^{\ln x} = x$ (Inverse Property)
4. If $\ln x = \ln y$, then $x = y$ (One-to-One Property)

Evaluate the following:

$\ln \frac{1}{e^2} =$	$3\ln e =$	$e^{\ln 6} =$

Earlier we said that the domain of a log function is restricted to positive real numbers. This makes for a third consideration when finding the domain of a function (the first two being no division by zero and no negatives under radicals.)

Find the domain for the following.

$f(x) = \ln(x+5)$
$f(x) = \ln(x^2 - 4)$
$f(x) = \ln(x^4)$

Please note since this is often a new topic for some students I highly encourage you to do more problems from the homework as extra practice until you understand logs.