MA 180 Lecture
Chapter 2
College Algebra and Calculus by Larson/Hodgkins
Functions and Graphs

2.8) Inverse Functions

Since a function can be represented by the collection of ordered pairs satisfying the equation, we can reverse the ordered pairs to create another relation called the inverse relation. Only when that inverse relation is a function do we say the function has an **inverse function**. We may arrive at this by restricting the domain of the inverse relation to create a function. We use the following notation, for a function *f*, we say the inverse function is f^{-1}.

It is true then that if we apply a function followed by its inverse function, we arrive back to the value we started with. We can think of an inverse function as un-doing what the function did.

Thus $\left(f \circ f^{-1}\right)(x) = \left(f^{-1} \circ f\right)(x) = x$.

Some common examples of inverse functions would be the following: if $f(x) = x + 5$ then $f^{-1}(x) = x - 5$ since the way to undo *adding* five is by *subtracting* five. Another example would be $f(x) = x^2$ and $f^{-1}(x) = \sqrt{x}$ but only if we restrict the domain on *f* to be the set of non-negative real numbers.

Definition of Inverse Functions
Let *f* and *g* be two functions such that $f(g(x)) = x$ for every *x* in the domain of *g* and $g(f(x)) = x$ for every *x* in the domain of *f.* Under these conditions, the function *g* is the **inverse function** of the function *f*. The function *g* is denoted by f^{-1} (read "*f* inverse"). So $\left(f \circ f^{-1}\right)(x) = x$ and $\left(f^{-1} \circ f\right)(x) = x$. The domain of *f* must be equal to the range of f^{-1}, and the range of *f* must be equal to the domain of f^{-1}.
Note: f^{-1} means the inverse function, it does not mean the reciprocal (like negative exponents mean in other parts of algebra.)

Finding Inverse Functions
1. In the equation $f(x)$, replace $f(x)$ by *y*.
2. Interchange the roles of *x* and *y*.
3. Solve the new equation for *y*. If the new equation does not represent *y* as a function of *x*, the function *f* does not have an inverse function. If the new equation does represent *y* as a function of *x*, continue to step 4.
4. Replace *y* by $f^{-1}(x)$ in the new equation.
5. Verify that *f* and f^{-1} are inverse functions of each other by sowing that the domain of *f* is equal to the range of f^{-1}, the range of *f* is equal to the domain of f^{-1}, and $f^{-1}(f(x)) = x = f(f^{-1}(x))$.

Examples: Find f^{-1}.

$$f(x)=\frac{3x-5}{7}$$

$$f(x)=\sqrt[3]{x+2}$$

$$f(x)=\frac{5x-1}{2x+3}$$

The graph of an inverse function is going to be symmetric with respect to the line *y=x*.

Horizontal Line Test for Inverse Functions

A function *f* has an inverse function if and only if no *horizontal* line intersects the graph of *f* at more than one point.

Consider the function $f(x) = x^2$. What does the graph say about this function having an inverse function? Is there a way to restrict the domain to remedy this?

Exercises:

Verify that the following are inverse functions of one another. $f(x) = \frac{1}{x+1}$ and $f^{-1}(x) = \frac{1-x}{x}$
Find the inverse function for $f(x) = (x-5)^2$