MA 180 Lecture Chapter 0 College Algebra and Calculus by Larson/Hodgkins Fundamental Concepts of Algebra

Section 0.7) Fractional Expressions

Part One

Domain of an Expression: The set of all real numbers for which an algebraic expression is defined. For now, the two rules we cannot violate are

- a.) No division by zero
- b.) No even roots of negative numbers

Two algebraic expressions are **equivalent** if they have the same domain and yield the same values for all numbers in their domain. Eg. [(x-2)+(x+5)] and 2x+3 are equivalent.

Examples:

Find the domain of the following algebraic expressions.

Expression	Domain
$x^5 - 7x^2 + 3$	Polynomials
$x^2 + 3x - 2, x > 0$	Restricted domain given
$\sqrt{x-2}$	Even Radicals
$\frac{x+5}{x-7}$	Division by zero

Notation

 $\Re{2,5}$ means all real numbers except (minus) the numbers two and five.

The quotient of two algebraic expressions is a **fractional expression**. The quotient of two polynomials is a **rational expression**. As with fractions we want all fractional expressions in simplest terms so we factor out then divide (cancel) common factors.

Example:

$$\frac{x^2 - 3x}{x^2 + 5x} = \frac{x(x - 3)}{x(x + 5)} = \frac{x - 3}{x - 5}$$

Please note the Domain for this is all real numbers except zero and five. This domain restriction must be made so the two fractions are equivalent.

Simplifying a Rational Expression

Example

$$\frac{x^2 - 7x + 12}{x^3 - 4x^2} = \frac{(x - 3)(x - 4)}{x^2(x - 4)} = \frac{x - 3}{x^2}$$

$$\frac{x^3 - 27}{x^2 + 3x + 9} =$$

$$\frac{2 - x + 2x^2 - x^3}{x - 2} =$$

Exercise: Simplify the following.

$$\frac{x^2 + 8x - 20}{x^2 + 11x + 10} =$$

$$\frac{x^2 - 25}{5 - x} =$$

Multiplying Rational Expressions

Multiply numerators and denominators and cancel common factors.

Examples:

$$\frac{5}{x-1} \cdot \frac{x-1}{25(x-2)} =$$

Note in the above example $x \neq 1, x \neq 2$

$$\frac{(x-9)(x+7)}{x+1} \cdot \frac{x}{9-x} =$$

Dividing Rational Expressions

When dividing you must first find the reciprocal (flip) the second fraction then multiply.

Examples:

$$\frac{x+2}{5(x-3)} \div \frac{x-2}{5(x-3)} =$$

$$\frac{\frac{x^2}{(x+1)^2}}{\frac{x}{(x+1)^3}} =$$

Exercises. Perform the indicated operation and simplify.

$\frac{4y-16}{2y+6} =$		
5y+15 $4-y$		
2		
$\frac{x^2 - x - 6}{2} \div \frac{x^2 - 4}{2} =$		
$x^2 + 6x + 9 \qquad x + 3$		

Homework for Section 0.7 Part One Pages 60-6 1 # 2,3,7,8,9,19,22,29,32,39,42,47,48