MA 180 Lecture
Chapter 0
College Algebra and Calculus by Larson/Hodgkins
Fundamental Concepts of Algebra

0.3) Integer Exponents

Let *a* be a real number, a variable, or an algebraic expression, and let *n* be a positive integer. Then $a^n = a \cdot a \cdot a \cdot \ldots \cdot a$ where the right hand side is the multiplication of *a* a total of *n* times. We say *n* is the **exponent** and *a* is the **base**.
The expression a^n is read as "a to the n^{th} **power**" or simply "a to the n^{th}."

Properties of Exponents

Let a and *b* be real numbers, variables, or algebraic expressions, and let *m* and *n* be integers. (Assume all denominators and bases are nonzero.)

Property	Example	Description
1.) $a^m a^n = a^{m+n}$	$3^2 3^5 =$	Product of Powers
2.) $\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{4^7}{4^5} =$	Quotient of Powers
3.) $(ab)^m = a^m b^m$	$(2x)^3 =$	Power of a Product
4.) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\left(\dfrac{3}{5}\right)^2 =$	Power of a Quotient
5.) $(a^m)^n = a^{mn}$	$(7^4)^5 =$	Power of a Power
6.) $a^{-m} = \dfrac{1}{a^m}$	$6^{-3} =$	Definition of negative exponent
7.) $a^0 = 1, a \neq 0$	$978^0 =$	Definition of zero exponent
8.) $\left(\dfrac{a}{b}\right)^{-m} = \left(\dfrac{b}{a}\right)^m$	$\left(\dfrac{2}{3}\right)^{-2} =$	
9.) $\lvert a^2 \rvert = \lvert a \rvert^2 = a^2$	$\lvert 6^2 \rvert =$	

Please Note

There is a difference between $-2x^4$ and $(-2x)^4$. In the first example only the variable part is being raised to a power. In the second example, everything inside the parenthesis is being raised to the power. So the second is equivalent to $(-2x)^4 = (-1)^4(2)^4(x)^4 = +16x^4$.

Exercise:

Evaluate the expression. Write fractional answers in simplest form. Use only positive exponents when expressing exponents.

1.) $2^5 2^{-3} =$
2.) $-5^2 =$
3.) $(-5)^2 =$
4.) $\left(\frac{1}{2}\right)^{-3} =$
5.) $\frac{x^6}{2x^3} =$
6.) $\left(-\frac{3}{5}\right)^3\left(\frac{5}{3}\right)^2 =$
7.) $8x^0 - (8x)^0 =$
8.) $(6y^2)(2y^3)^3 =$
9.) $\frac{35y^{10}}{7y^5} =$
10.) $24 \cdot \frac{(x-7)^3}{8(x-7)} =$
11.) $\frac{x^n \cdot x^{2n}}{x^{3n}} =$
12.) $4^n \cdot 2^{2n} =$
13.) $(y+2)^{-3}(y+2)^{-1} =$
14.) $(-2x^2)^3(4x^3)^{-1} =$
15.) $\frac{x^3y^{-2}z}{x^4y^5z^{-6}} =$

Scientific Notation:

Scientific Notation has the form $c \times 10^n$, where $1 \le c < 10$ and *n* is an integer. The *positive* exponent *n* indicates the number is large and that the decimal point has been moved *n* places. A *negative* exponent in scientific notation indicates that the number is *small* (less than one) and has *n* spaces between decimal places.

Examples:

$$0.00000365 = 3.65 \times 10^{-6}$$

$$109{,}000{,}000 = 1.09 \times 10^{8}$$

Applications:

The balance *A* in an account that earns an annual interest rate *r* (in decimal form) for *t* years is given by one of the following:

$A = P(1 + rt)$ Simple Interest

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound Interest

In both formulas, *P* is the principal (or the initial deposit.) In the formula for compound interest, *n* is the number of compounding *per year*. Make sure you convert all units of time *t* to years. For instance, six months is one half year, so $t = \frac{1}{2}$.

Exercises:

You want to invest $1000 for four years. Which savings plan will earn more money? A.) 5% simple annual interest or B.) 4.5% interest compounded quarterly