MA 180 Lecture
Chapter 1
College Algebra and Calculus by Larson/Hodgkins
Equations and Inequalities

1.4) The Quadratic Formula

In order to use the quadratic formula to solve any quadratic equation we must first derive the formula. We do this from the method of completing the square. The following are instructions for completing the square. It is most likely easier to do a few examples. *** Please note that the method in the book is not the same method that I teach as there are many variations on completing the square. The method described below is useful in more applications than just solving quadratics. It is a more generic way to find the completed square.

Complete the square

a. Note this is not an equation, we cannot divide by "a" (but if you must, please remember to multiply it by "a" when you are done.)
b. Given $ax^2 + bx + c$
c. First factor and add parenthesis $a(x^2 + \frac{b}{a}x) + c$
d. Calculate $\left(\frac{b}{2a}\right)^2$
e. Add and subtract within the parenthesis $a(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2) + c$
f. Remove the latter from parenthesis and distribute
   $a(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2) - \frac{b^2}{4a} + c$
g. Factor to complete the square $a(x + \frac{b}{2a})^2 - \frac{b}{4a}^2 + c$
h. Note if $ax^2 + bx + c = f(x)$, then $f(-\frac{b}{2a}) = -\frac{b}{4a}^2 + c$

Example:

Complete the square:

$$x^2 - 6x + 9$$

Complete the square:

$$\begin{aligned}
&x^2 - 6x + 10 \\
&= (x^2 - 6x) + 10 \\
&= (x^2 - 6x + 9 - 9) + 10 \\
&= (x^2 - 6x + 9) - 9 + 10 \\
&= (x - 3)^2 + 1
\end{aligned}$$

Complete the square:

$$x^2 + 16x - 5$$

Complete the square:

$$x^2 - 24x + 7$$

Complete the square:

$$2x^2 - 20x + 9$$

Complete the square:

$$-3x^2 + 30x - 4$$

If we take the completed square from the above process we get $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + c$.

So if we set it equal to zero and solve, then we will get the following.

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + c = 0$$

This is our well known **quadratic formula.**

For $ax^2 + bx + c = 0$ the solutions is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In the quadratic formula, the quantity under the radical sign, $b^2 - 4ac$, is called the **disciminant**. It can be used to determine the number of solutions of a quadratic equation.

Solutions of a Quadratic
The solutions of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ can be classified by the discriminant, $b^2 - 4ac$, as follows:
1. If $b^2 - 4ac > 0$, the equation has *two* distinct real solutions.
2. If $b^2 - 4ac = 0$, the equation has *one* distinct real solutions.
3. If $b^2 - 4ac < 0$, the equation has *no* distinct real solutions.

Determine how many solutions the following quadratic equations have.

$x^2 - 3x + 4 = 0$
$2x^2 - 4x - 5 = 0$
$x^2 - 4x + 4 = 0$

Use the quadratic formula to solve the following:

$2x^2 + x - 1 = 0$

$12x - 9x^2 = -3$
$3x^2 - 16 = 38$
$x^2 - 8x + 16 = 0$
$x^2 + 3x + 1 = 0$