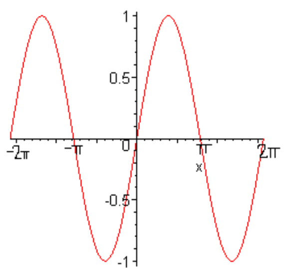
MA 131 Lecture Notes
Sections 1.3

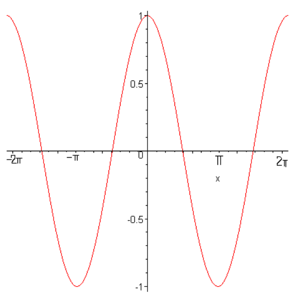
Here are a few more basic (library) functions. We will discuss exponential, logarithmic, and trigonometric functions in detail later.

$f(x) = \sin x$



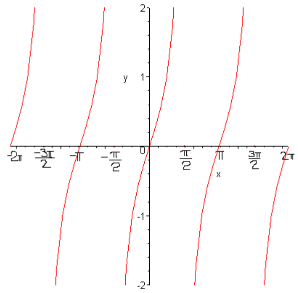
$f(x) = \cos x$

The graph of $y = \cos x$ is a horizontal shift of the graph of $y = \sin x$.



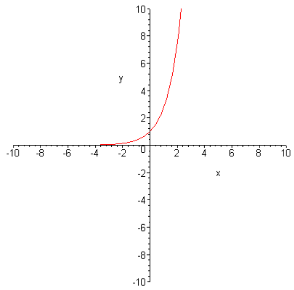
$f(x) = \tan x$

Remember $\tan x = \dfrac{\sin x}{\cos x}$.



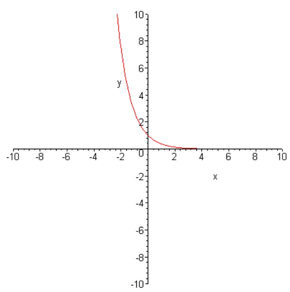
$y = e^x$

The graph of that $y = e^x$ will be similar in structure to any graph of the form $y = a^x$ where $a > 1$.



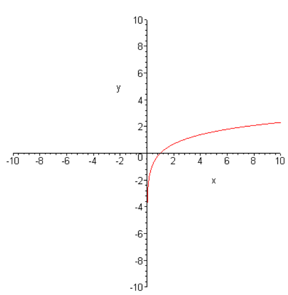
$y = e^{-x}$

The graph of that $y = e^{-x}$ will be similar in structure to any graph of the form $y = a^x$ where $0 < a < 1$.



$y = \ln x$

The graph of $y = \ln x$ is the inverse of $y = e^x$ so it will be a reflection across the line $y = x$ and will also be similar to any function $y = \log_a x$ where $a > 1$.



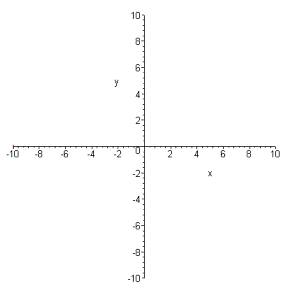
Once you've memorized the library of functions above, you can use transformations to create new graphs.

Let's start with something we are very familiar with; the graph of a line. If asked to graph $y = 2x - 5$ we would note that it was in the form $y = mx + b$. We recognize that $m = 2$ corresponds to the slope of the line and $b = -5$ is the *y*-intercept. We could graph this with the knowledge of slopes and intercepts or we could recognize this as a translation. We start with the base function $y = x$ and shift the graph down five units then vertically stretching the graph by a factor of 2.

Here are the transformation rules and examples of each.

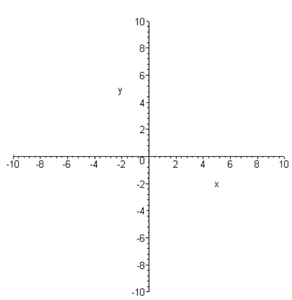
Vertical Shifts of Graphs
Suppose $c > 0$
To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upwards *c* units.
To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downwards *c* units.

Ex. Graph $y = x^2 + 5$



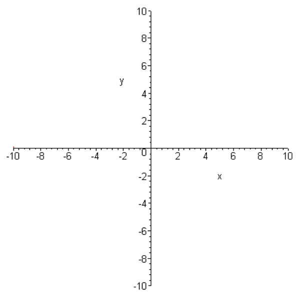
Horizontal Shifts of Graphs
Suppose $c > 0$
To graph $y = f(x - c)$, shift the graph of $y = f(x)$ to the right *c* units.
To graph $y = f(x + c)$, shift the graph of $y = f(x)$ to the left *c* units.

Ex. Graph $y = (x-3)^2$ and $y = (x-3)^2 + 5$ on the same axis.



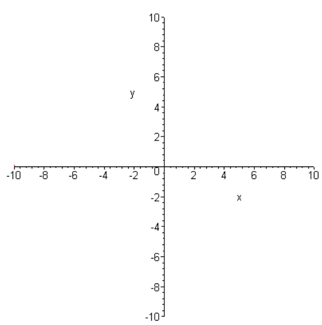
Reflecting Graphs
To graph $y = -f(x)$, reflect the graph of $y = f(x)$ in the *x*-axis.
To graph $y = f(-x)$, reflect the graph of $y = f(x)$ in the *y*-axis.

Ex. Graph $y = -|x-2|$



Vertical Stretching and Shrinking of Graphs
To graph $y = cf(x)$:
If $c > 1$, stretch the graph of $y = f(x)$ vertically by a factor of *c.*
If $0 < c < 1$, shrink the graph of $y = f(x)$ vertically by a factor of *c.*

Ex. Graph $y = 3x^2 - 1$



Horizontal Stretching and Shrinking of Graphs
To graph $y = f(cx)$:
If $c > 1$, shrink the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$.
If $0 < c < 1$, stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$.

Homework:
Section 1.1 #5,6,23,27,30,65-70
Section 1.2 #1,4
Section 1.3 #3,11,16,17