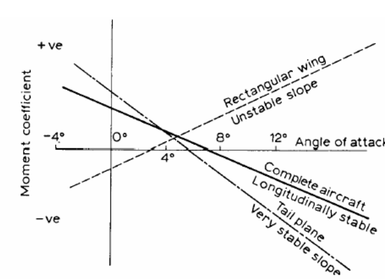
AE 430 - Stability and Control of Aerospace Vehicles

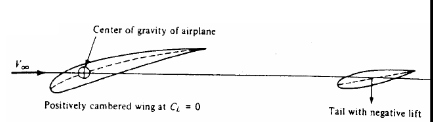
Longitudinal Static Stability

Tailplanes

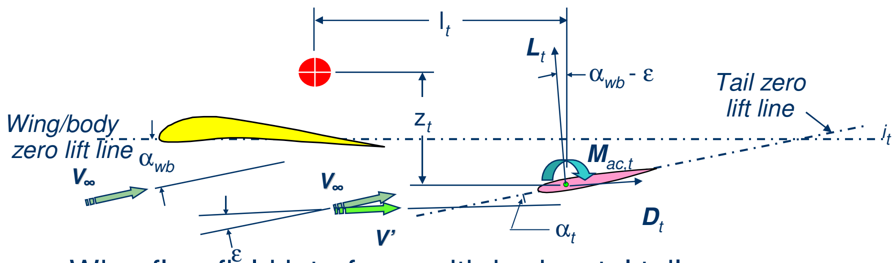
- **Typically a wing alone will have –ve C_{M0} (C_m at zero lift) (positive camber airfoil)**
- **It may have a** $$\frac{dC_{mG}}{d\alpha} > 0$$ **if CG is aft of AC**
- **By positioning a tailplane *behind* the wing and inclining it to give –ve lift**
  - It improves the stability of the wing-tail combination
- **-ve lift reduces overall lift of aircraft, so wings must carry more lift**
- **Results in increase of induced drag, plus drag of tailplane – Trim Drag**

Usually symmetrical airfoil are used for the tail because must produce both upward and downward airloads





Wing-Horizontal Tail Geometry



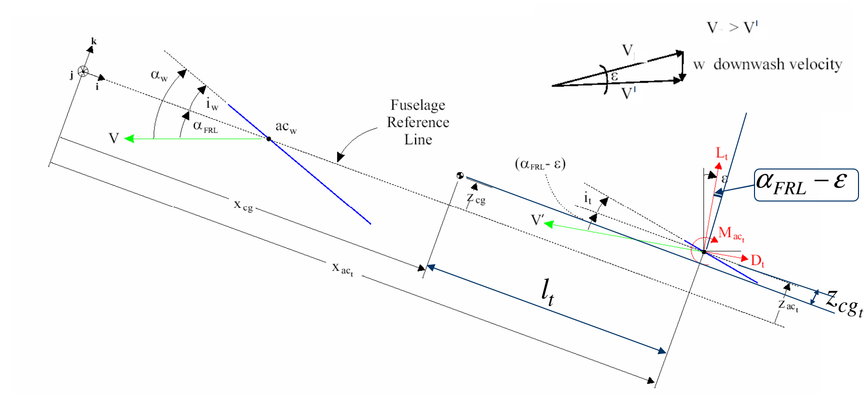
- Wing flow field interferes with horizontal tail
  - downwash deflects V_∞ downward
  - local relative wind is reduced in magnitude; tail "sees" lower dynamic pressure
- Fuselage blanks out part of the tail

Downwash ε

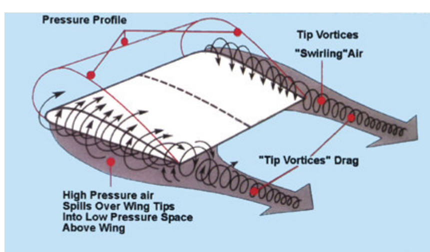
- The value of the downwash at the tail is affected by fuselage geometry, flap angle wing platform, and tail position. It is best determined by measurement in a wind tunnel, but lacking that, lifting surface computer programs do an acceptable job. For advanced design purposes it is often possible to approximate the downwash at the tail by the downwash far behind an elliptically-loaded wing:

$$\varepsilon \approx \frac{2C_{L_w}}{\pi AR_w} \qquad \text{So,} \qquad \frac{\partial \varepsilon}{\partial \alpha} \approx \frac{2C_{L_{\alpha w}}}{\pi AR_w}$$

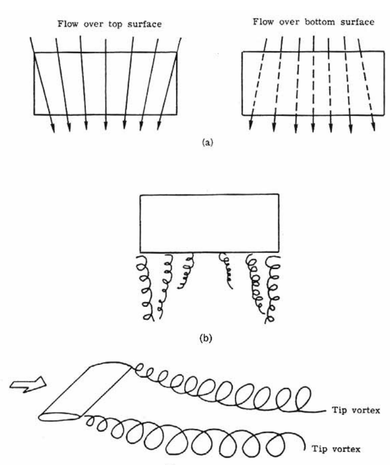
Tail Contribution – Aft Tail

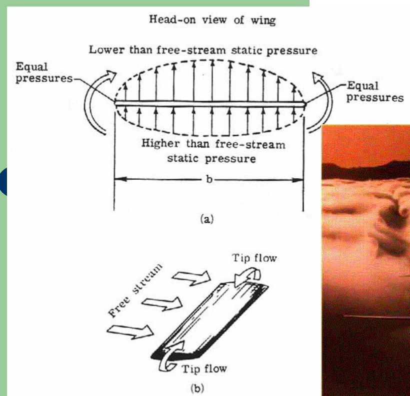


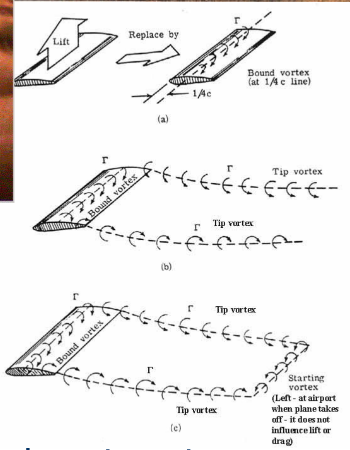
Wing-tip vortex



Wing-tip vortices are formed when high-pressure air spills up over the wing tips into the low-pressure space above the wing.

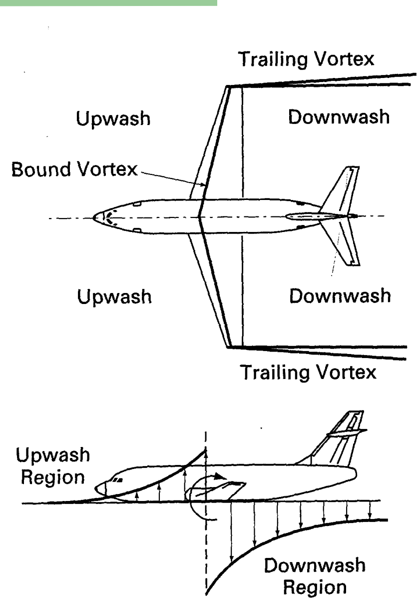


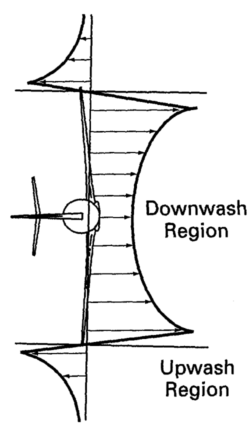




Pressures must become equal at the wing tips since pressure is a continuous function (figure a). The free stream flow combines with tip flow, resulting in an inward flow of air on the upper wing surface and an outward flow of air on the lower wing surface (figure b).

Complete-wing vortex system.





Flow field around an airplane created by the wing

Tail Contribution – Aft Tail

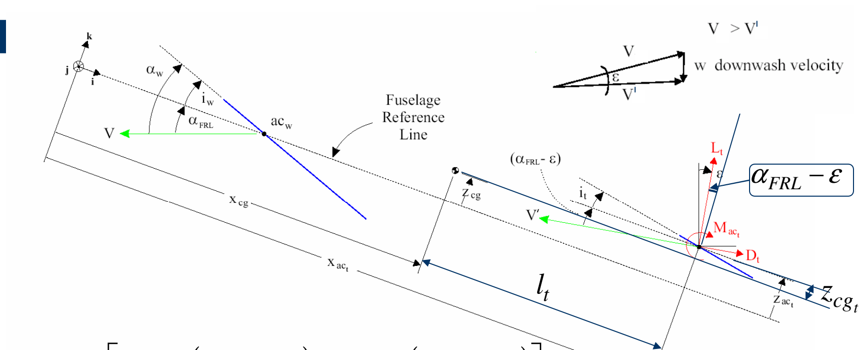
$\alpha_t = \alpha_w - i_w - \varepsilon + i_t$ **Angle of attach at the tail**

$L = L_w + L_t$ **Total lift wing-tail configuration**

$$C_L = C_{Lw} + \frac{\frac{1}{2}\rho V_t^2 S_t}{\frac{1}{2}\rho V_w^2 S} C_{Lt} = C_{Lw} + \eta \frac{S_t}{S} C_{Lt}$$

$$\eta = \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V_w^2} = \frac{Q_t}{Q_w}$$ **Tail efficiency – ratio of dynamic pressure**

Tail Contribution – Aft Tail



$$M_t = -l_t\left[L_t \cos(\alpha_{FRL} - \varepsilon) + D_t \sin(\alpha_{FRL} - \varepsilon)\right]$$
$$+ z_{cg_t}\left[D_t \cos(\alpha_{FRL} - \varepsilon) - L_t \sin(\alpha_{FRL} - \varepsilon)\right] + M_{ac_t}$$

$$M_t = -l_t L_t = -l_t \tfrac{1}{2}\rho V_t^2 S_t C_{L_t} = -l_t Q_t S_t C_{L_t}$$

Tail Contribution – Aft Tail

$$M_{cg\,t} = -l_t L_t = -l_t \tfrac{1}{2}\rho V_t^2 S_t C_{L_t} = -l_t Q_t S_t C_{L_t}$$

$$C_{m_{cg\,t}} = \frac{M_{cg_t}}{\frac{1}{2}\rho V^2 S\bar{c}} = -l_t \frac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V^2}\frac{S_t}{S\bar{c}} C_{L_t} = -l_t \eta \frac{S_t}{S\bar{c}} C_{L_t} = -V_H \eta C_{L_t}$$

$$V_H = l_t \frac{S_t}{S\bar{c}}$$ **Horizontal tail volume ratio**

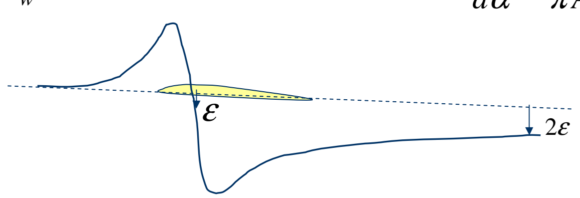
$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t = C_{L_{\alpha_t}} \left(\alpha_w - i_w - \varepsilon + i_t\right)$$

Downwash

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha_w$$

Downwash at zero angle of attach

$$\varepsilon = \frac{2C_{L_w}}{\pi AR_w} \quad (\mathrm{rad})$$ **Finite wing theory** $$\frac{d\varepsilon}{d\alpha} = \frac{2C_{L_{\alpha_w}}}{\pi AR_w}$$



Tail Contribution – Aft Tail

$$C_{m_{cg\,t}} = -V_H \eta C_{L_t}$$

$$C_{m_{cg\,t}} = -V_H \eta C_{L_{\alpha t}} \left(\alpha_w - i_w - \varepsilon + i_t\right)$$

$$C_{m_{cg\,t}} = \underbrace{V_H \eta C_{L_{\alpha t}} \left(\varepsilon_0 + i_w - i_t\right)}_{C_{m_{0t}}} \underbrace{- V_H \eta C_{L_{\alpha t}} \left(1 - \frac{d\varepsilon}{d\alpha}\right)}_{C_{m_{\alpha t}}} \alpha_w$$

$$V_H = l_t \frac{S_t}{S\bar{c}}$$

$$C_{m_{cg\,t}} = C_{m_{0t}} + C_{m_{\alpha t}} \alpha_w$$

To have $C_{m_{0t}} > 0$ we can adjust the tail incidence angle i_t

$C_{m_{\alpha t}} < 0$ we can select properly V_H and $C_{L_{\alpha t}}$ to stabilize the aircraft

($C_{L\alpha t}$ can be increased for example, by increasing the aspect ratio of the tail)

Example # 3

- Consider the wing-body model in Example # 2 (previous class). Assume that a horizontal tail with no elevator is added to this model. The wing area and chord are 1.5 m^2 and 0.45 m, respectively.
- The distance from the airplane's center of gravity to the tail's aerodynamic center is 1.0 m. The area of the tail is 0.4 m^2, and the tail-setting angle is - 2.0°. The lift slope of the tail is 0.12 per degree.
- From experimental measurement, ε_0= 0 and $d\varepsilon/d\alpha$ = 0.42. If the absolute angle of attack of the model is 5° and the lift at this angle of attack is 4134 N, calculate the moment about the center of gravity.

$$q_\infty = 6125 N/m^2; \qquad C_{m_{ac,w}} = -0.003; \qquad i_w = 0$$

$$\frac{x_{cg}}{\bar{c}} - \frac{x_{ac,w}}{\bar{c}} = 0.02; \qquad \eta = 1 \text{ (assumed)}$$

$$C_{Lw} = \frac{L}{q_\infty S} = \frac{4134}{6125*1.5} = 0.45$$

$$C_{L_{\alpha,w}} = \frac{dC_{Lw}}{d\alpha} = \frac{0.45}{5} = 0.09$$

$$V_H = \frac{l_t S_t}{\bar{c} S} = \frac{1.0*0.4}{0.45*1.5} = 0.593$$

$$C_{mcg} = C_{mac,wb} + C_{L\alpha,wb}\alpha_{wb}\left[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac,wb}}{\bar{c}} - V_H \frac{C_{L\alpha,t}}{C_{L\alpha,wb}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]$$
$$+ V_H C_{L\alpha,t}\left(\varepsilon_0 - i_t\right)$$

$$C_{mcg} = -0.003 + 0.09*5\left[(0.02) - 0.593\frac{0.12}{0.09}(1 - 0.42)\right]$$
$$+ 0.593*0.12(2) = -0.058$$

$$M_{cg} = q_\infty S \bar{c} C_{mcg} = 6125*1.5*0.4*(-0.058) = -240 N/m$$

Example # 4

- Consider the wing-body-tail model of Example # 3. Does this model have longitudinal static stability and balance?

$$\frac{\partial C_{mcg}}{\partial \alpha_{wb}} = C_{L\alpha,wb}\left[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac,wb}}{\bar{c}} - V_H \frac{C_{L\alpha,t}}{C_{L\alpha,wb}}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right]$$

Negative slope; OK

$$= 0.09\left[0.02 - 0.593\frac{0.12}{0.09}(1 - 0.42)\right] = -0.039$$

$$C_{m,0} = C_{mac,wb} + V_H C_{L\alpha,t}\left(\varepsilon_0 - i_t\right)$$
$$= -0.003 + 0.593*0.12*(2.0) = 0.139$$

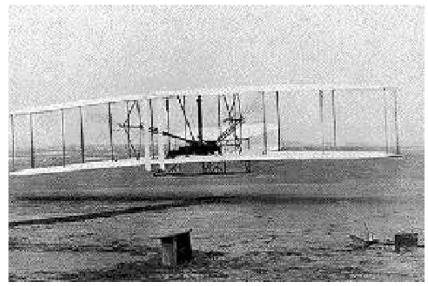
Positive $C_{m,0}$; OK

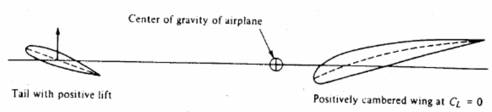
$$C_{m,cg} = 0.139 - 0.039\alpha_e = 0$$
$$\alpha_e = 3.56^0$$

Equilibrium angle-of-attack; reasonable; OK

Canard

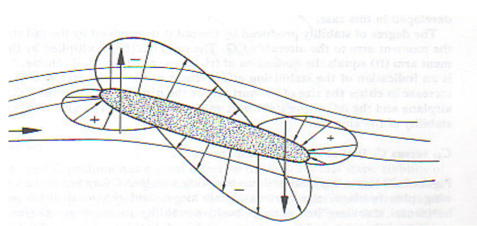
- Alternatively put the *tail* ahead of the wing – known as foreplanes or canards
- Canard needs to generate +ve lift to create +ve pitching moment about CG
- This means that canard contributes to overall lift of aircraft
- Canard can create fast increase in lift, making aircraft very responsive
- Canard interferes with airflow over main wing – causes resultant force vector of wing to *tilt* backward – increasing drag





Fuselage Contribution

- Streamlined fuselage has pressure distribution similar to body of revolution
- No net lift developed by pressure distribution
- Nose-up pitching moment developed by up-gust
- Pitching moment is destabilizing because not countered by net lift vector
- **Fuselage is destabilizing component**



Fuselage Contribution

$$\frac{dM}{d\alpha} = \text{fn}\left(\text{Vol},\, Q = \tfrac{1}{2}\rho V^2\right)$$

$$C_{m_{0_f}} = \frac{k_2 - k_1}{36.5 S\bar{c}} \int_0^{l_f} w_f^2 \left(\alpha_{0_w} + i_f\right) dx$$

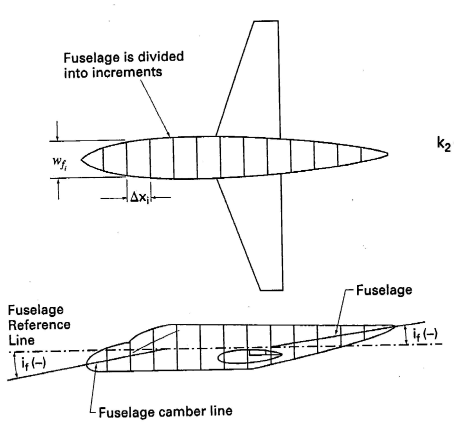
$$C_{m_{0_f}} = \frac{k_2 - k_1}{36.5 S\bar{c}} \sum_{x=0}^{x=l_f} w_f^2 \left(\alpha_{0_w} + i_f\right) \Delta x$$

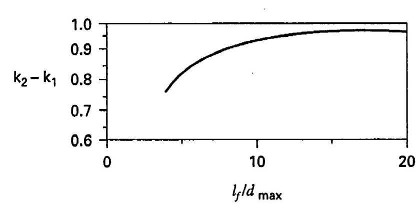
$$C_{m_{\alpha_f}} = \frac{1}{36.5 S\bar{c}} \int_0^{l_f} w_f^2 \frac{\partial \varepsilon_u}{\partial \alpha} dx \qquad \left(\text{deg}^{-1}\right)$$

$$C_{m_{\alpha_f}} = \frac{1}{36.5 S\bar{c}} \sum_{x=0}^{x=l_f} w_f^2 \frac{\partial \varepsilon_u}{\partial \alpha} \Delta x \qquad \left(\text{deg}^{-1}\right)$$

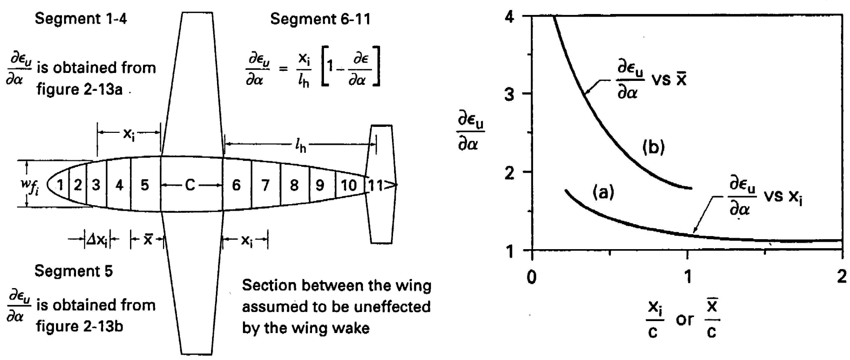
See textbook (pp. 53-55) for details

Fuselage Contribution





Fuselage Contribution



Fuselage contribution

- Gilruth (NACA TR711) developed an empirically-based method for estimating the effect of the fuselage:

$$\frac{\partial C_{m_{fuse}}}{\partial C_L} = \frac{K_f w_f^2 L_f}{S_w \bar{c} C_{L_{\alpha_w}}}$$

where:
$C_{L\alpha w}$ is the wing lift curve slope per radian
L_f is the fuselage length
w_f is the maximum width of the fuselage
K_f is an empirical factor discussed in NACA TR711 and developed from an extensive test of wing-fuselage combinations in NACA TR540.

Position of 1/4 root chord on body as fraction of body length	Kf
.1	.115
.2	.172
.3	.344
.4	.487
.5	.688
.6	.888
.7	1.146

K_f is found to depend strongly on the position of the quarter chord of the wing root on the fuselage. In this form of the equation, the wing lift curve slope is expressed in rad^{-1} and K_f is given in the table. (Note that this is not the same as the method described in Perkins and Hage.) The data shown in table were taken from TR540 and Aerodynamics of the Airplane by Schlichting and Truckenbrodt:

Power Effects

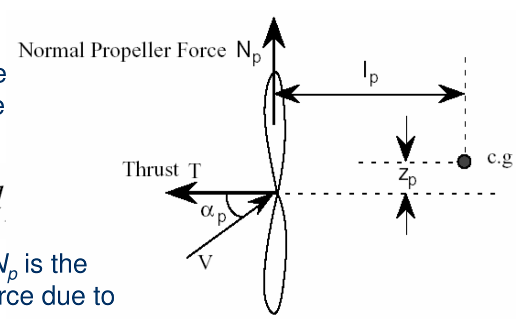
- Propeller
- Jet Engine

Both will produce a contribution to C_{m_α}

Propulsive System Contribution

The incremental pitching moment about the airplane center of gravity due to the propulsion system is:

$$\Delta M_{cg} = Tz_p + N_p l_p$$



where T is the thrust and N_p is the propeller or inlet normal force due to turning of the air.

Another influence comes from the increase in flow velocity induced by the propeller or the jet slipstream upon the tail, wing and aft fuselage.

Propulsive System contribution

In terms of moment coefficient,

$$\Delta C_{M_{cg}} = \frac{T}{qS}\frac{z_p}{c} + \frac{N_p}{qS}\frac{l_p}{c}$$

Since the thrust is directed along the propeller axis and rotates with the airplane, its contribution to the moment about the center of gravity is independent of α_w. Then we have

$$\Delta C_{M_o} = \frac{T}{qS}\frac{z_p}{c}$$

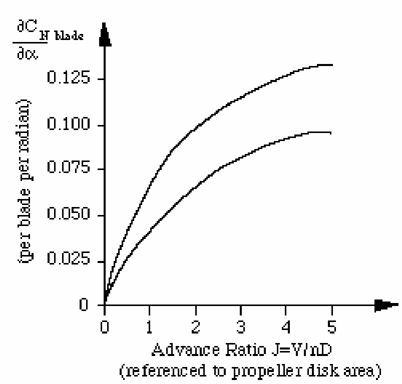
$$N_p = N_{prop} q S_p C_{N_p\alpha}(1-\varepsilon_\alpha)\alpha$$

$$\Delta C_{M_\alpha} = N_{prop}\frac{S_{prop} l_p}{Sc}\frac{\partial C_{N_p}}{\partial \alpha}(1-\epsilon_\alpha)$$

where the propeller normal force coefficient $\partial C_{N_p}/\partial\alpha$ and the downwash (or upwash) ε_α are usually determined empirically

Propulsive System contribution

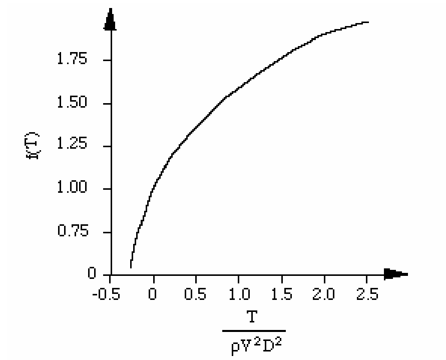
- N_{prop} is the number of propellers and S_{prop} is the propeller disk area ($\pi D^2/4$) and D is the diameter of the propeller. Note that a propeller mounted aft of the c.g. is stabilizing.
- This is one of the advantages of the pusher-propeller configuration. Note that n in is the propeller angular speed in *rps* .



Propulsive System contribution

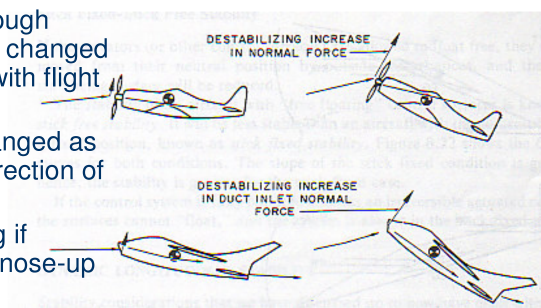
Propeller normal force coefficient

$$C_{N_{p\alpha}} = \frac{\partial C_{N_{blade}}}{\partial \alpha} f(T)$$



Engine Nacelles

- Direction of airflow through propeller or engine not changed if engine axis aligned with flight path
- Direction of airflow changed as necessary to flow in direction of engine axis
- Side force destabilizing if resulting force causes nose-up pitching moment



- Stabilizing if resulting force causes nose-down pitching moment
- Propellers/ jet engine intake behind CG are stabilizing

Stick Fixed Neutral Point

$$C_{m_{cg}} = C_{m_0} + C_{m_\alpha}\alpha_w$$

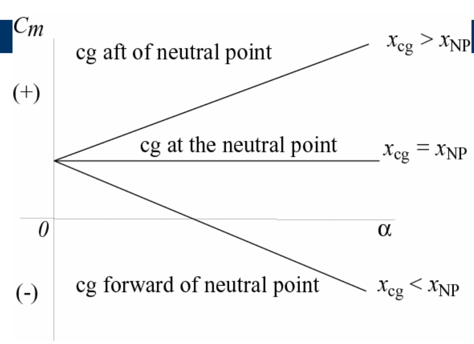
where, for a wing-tail-fuselage configuration (see also previous examples):

$$C_{m_0} = C_{m_{0w}} + C_{m_{0f}} + C_{m_{0t}}$$

$$= C_{m_{0w}} + C_{m_{0f}} + V_H \eta C_{L_{\alpha_t}}\left(\varepsilon_0 + i_w - i_t\right)$$

$$C_{m_\alpha} = C_{L_{\alpha w}}\left[\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right] + C_{m_{\alpha f}} - V_H \eta C_{L_{\alpha_t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

Stick Fixed Neutral Point



$$C_{m_\alpha} = 0$$

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{C_{m_{\alpha f}}}{C_{L_{\alpha w}}} + V_H \eta \frac{C_{L_{\alpha t}}}{C_{L_{\alpha w}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

Trim and Neutral Point

- Trim

$$C_{m_{cg}} = C_m\big|_{C_L=0} + \frac{dC_m}{dC_L} C_L \qquad C_{m_{cg}} = 0; \quad C_{L_{trim}} = -\frac{C_m\big|_{C_L=0}}{dC_m / dC_L}$$

- Neutral Point

For once the NP is know, the stability at any other c.g. position may be obtained with good accuracy from the following relation:

$$\frac{dC_m}{dC_L} = \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}} \right)$$

$$\left(\frac{x_{NP}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} \right)$$ Stick fixed static margin

Example # 5

- For the configuration of Examples # 3/4, calculate the neutral point and static margin x_{cg} = 0.26

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac,wb}}{\bar{c}} + V_H \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha w}}} \left(1 - \frac{\partial \varepsilon}{\partial \alpha} \right)$$

$$\text{as} \quad \frac{x_{cg}}{\bar{c}} - \frac{x_{ac,wb}}{\bar{c}} = 0.02;$$

$$\frac{x_{ac,wb}}{\bar{c}} = \frac{x_{cg}}{\bar{c}} - 0.02 = 0.26 - 0.02 = 0.24$$

$$\frac{x_{NP}}{\bar{c}} = 0.24 + 0.593 \frac{0.12}{0.09} (1 - 0.42) = 0.70$$

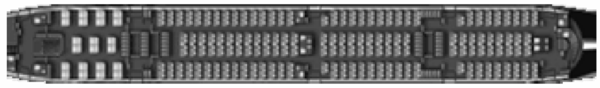
$$\text{static margin} = \frac{x_{NP}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} = 0.7 - 0.26 = 0.44$$

Some Conclusions

- For static stability the centre of gravity must be in front of the stick fixed neutral point
- When the centre of gravity reaches the stick fixed neutral point the aircraft is neutrally stable
- If the centre of gravity moves behind (closer to the tail) the stick fixed neutral point the aircraft becomes statically unstable

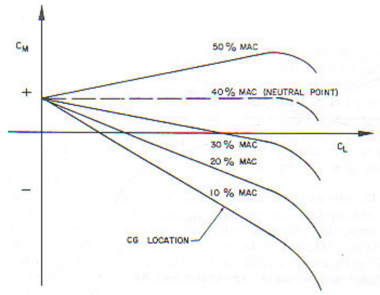
CG Movement

- **During flight the CG can move substantially**
- **As CG moves forward the aircraft becomes more stable**
  - The forward limit to CG position is limited by the moment that the tailplane can produce
  - This is a function of tailplane lift and the tailplane volume (tailplane moment arm times its area)
- **While stability improves with forward CG movement**
  - Drag increases, this increase is known as **Trim Drag**
  - Aircraft maneuverability can suffer, larger control movements are required, and response becomes sluggish
- **When CG moves backwards**
  - Aircraft eventually becomes unstable
  - Trim drag reduces
  - CG position when aircraft is on point of becoming unstable is known as the **Neutral Point – i.e. For longitudinal stability the CG must always be in front of the neutral point**



CG Limits

- **The distance between the neutral point and centre of gravity is known as the *CG Static Margin*.**
  – For longitudinal stability the CG margin must be **+ve**
- **The absolute limit for forward CG position is determined by aircraft handling being too sluggish to control effectively**
- **The absolute limit for rear CG position is the onset of instability, and aircraft handling being too sensitive to control**
- **Aircraft Designers and Regulatory Authorities impose a more restricted CG range in practice**
- **Care must be taken by aircraft operators during loading to make sure that the CG position stays within the safe range**



Unstable Aircraft

- **The advent of fly-by-wire computer control systems makes unstable aircraft feasible in practice**
  – Computer must make continuous tiny adjustments to keep the airplane controllable
- **Advantages are:**
  – Configure tailplane/canard to produce +ve lift
  – Lower trim drag with CG at/behind neutral point, improving *L/D*
  – Quicker response to control inputs
- **Disadvantage, if the computer control system fails the aircraft is unflyable**
  – Must have redundant systems as safety precaution

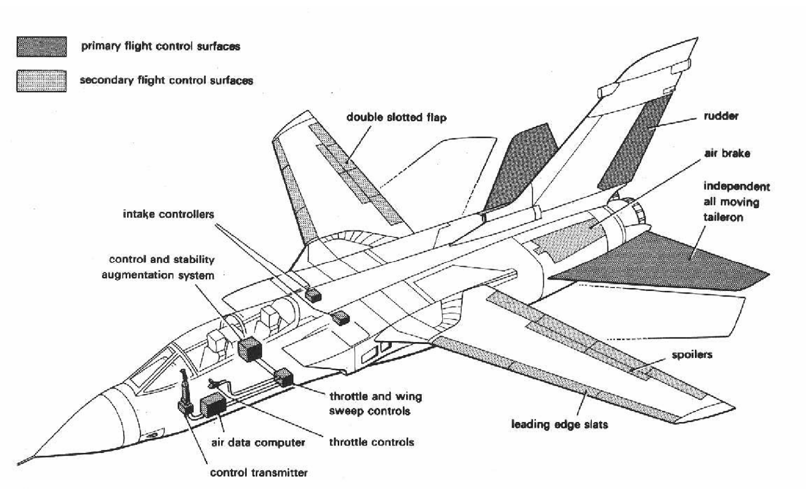
Static Stability Parameters

- Tailplane or canard design
- Aircraft Centre of Gravity position
- Main wing pitching moment characteristics
- Tail size- bigger tail means more lift/downforce
- Tail moment arm – further away from CG generates greater moment
- Tail angle of attack relative to main wing angle of attack – known as the longitudinal dihedral

Handling qualities are important

- Aircraft that are too stable are difficult to control
- Aircraft that tend towards instability can be difficult to control too

Control Systems



Influences on the Longitudinal Stability: Influence of Wing Flaps

Changes in the wing flaps affect both trim and stability. The main aerodynamic effects due to flap deflections are:

- Lowering the flaps has the same effect on $C_{mo;wb}$ as an increase in wing camber. That is producing a *negative* increment in $C_{mo;wb}$.
- The angle of wing-body zero-lift is changed to be more negative. Since the tail incidence i_t is measured relative to the wing-body zero lift line, this in effect places a positive increment in the tail incidence angle i_t .
- Change in the spanwise lift distribution at the wing leads to an increase in downwash at the tail, i.e. ε_o and $d\varepsilon / d\alpha$ may increase.

Effect of Airplane Flexibility

- Flexibility of an airframe under aerodynamic loads is evident in any flight vehicle. The phenomenon that couples aerodynamics with structural deformations is studied under the subject of **aeroelasticity**. There are two types of analysis:
  - Static behavior: Here the steady-state deformations of the vehicle structure are investigated. Phenomena such as aileron reversal, wing divergence and reduction in static longitudinal stability fall under this category.
  - Dynamic behavior: The major problem of interest is associated with the phenomena of dynamic loading, buffeting and flutter.