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Shear strain of a piezoelectric pipe induced by an electrode cover

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Abstract

Shear displacements and a related electric field induced in an elastic pipe via a film electrode cover are studied. The medium is a piezocrystal of the 6mm symmetry class. The problem is analyzed with the dual series equations involving trigonometric functions, which are reduced to the Fredholm integral equations of the second kind. The suggested algorithm is novel. This algorithm is efficient for solving the problem concerning pipes of an arbitrary thickness. The leading terms of the asymptotic expansions are found for a wide range of the actual parameters.

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This paper deals with an antiplane strain of an infinite pipe occupying the region $a \leq r \leq b$, $-\pi \leq \theta \leq \pi$, $-\infty < z < \infty$.

The strain state is induced by the constant electric field between the thin conductive films covering the inner surface of the pipe $r = a$, $-\pi \leq \theta \leq \pi$, $-\infty < z < \infty$ and the strip $-\alpha \leq \theta \leq \alpha$, $r = b$, $-\infty < z < \infty$ on the external surface (Fig. 1). The pipe material is a hexagonal crystal of the 6mm symmetry class (piezoceramics is polarized in the direction of the pipe axis chosen as z axis). The contacts are supposed to be ideally flexible.

There is no source of reference for the treated problem known to the author. The similar problem for a cylinder was studied in [1]. The technique used there is not effective for a thin pipe.

The state of the pipe is independent on the coordinate z. The electric field in free space is described with the harmonic potential $\varphi^*(r,\theta)$. The electric and elastic fields are determined by the electric potential $\varphi(r,\theta)$ and the displacement $u_z(r,\theta)$, which obey the equations [1]

$$\begin{aligned} \nabla^2 v &= 0\ ,\ \nabla^2\psi = 0,\ \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial\theta^2}, \\ v &= c_{44}^E u_z + e_{15}\varphi,\ \psi = e_{15}u_z - \backepsilon_{11}\,\varphi, \end{aligned} \tag{1}$$

where c_{44}^E is the elastic constant, e_{15} is the piezoelectric constant and $\backepsilon_{11}$ is the permittivity of the pipe material.

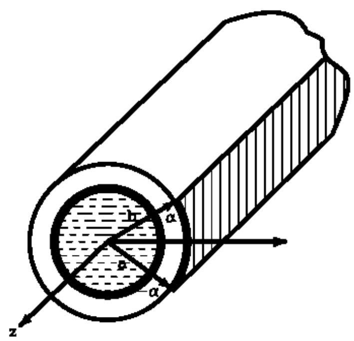

Figure 1: Geometry of the problem.

The nonzero components of the electric induction and shear stresses are expressed via the potentials $v(r,\theta)$ and $\psi(r,\theta)$:

$$\begin{aligned} D_r &= \frac{\partial\psi}{\partial r}\ ,\ D_\theta = \frac{1}{r}\frac{\partial\psi}{\partial\theta}; \\ \sigma_{\theta z} &= \frac{1}{r}\frac{\partial v}{\partial\theta}\ ,\ \sigma_{rz} = \frac{\partial v}{\partial r}. \end{aligned} \tag{2}$$

The stresses on the surface of the pipe are absent:

$$\frac{\partial v}{\partial r}\,|_{r=a} = \frac{\partial v}{\partial r}\,|_{r=b} = 0\ ,\quad -\pi \le \theta \le \pi,$$

which gives $v(r,\theta) = 0$. This implies that the stresses are zero everywhere in the pipe and the displacement, as well as the potential $\psi(r,\theta)$, are proportional to the electric potential $\varphi(r,\theta)$

$$u_z(r,\theta) = -\frac{e_{15}}{c_{44}^E}\varphi(r,\theta)\ ,\ \psi(r,\theta) = -\frac{e_{15}^2 + c_{44}^E\ \backepsilon_{11}}{c_{44}^E}\varphi(r,\theta). \tag{3}$$

The electric potential obeys the Laplace equation and satisfies the boundary conditions:

$$
\begin{aligned}
\varphi(a,\theta) &= 0\ ,\ -\pi \le \theta \le \pi; && (4)\\
\varphi(b,\theta) &= \varphi^*(b,\theta)\ ,\ \ -\pi \le \theta \le \pi; && (5)\\
\varphi(b,\theta) &= V_0\ ,\ \theta \in [-\alpha,\alpha]\ ; && (6)\\
D_r(b,\theta) &= D_r^*(b,\theta)\ \ ,\ \theta \notin [-\alpha,\alpha]\,, && (7)
\end{aligned}
$$

where $D_r^* = -\varepsilon_0 \frac{\partial \varphi^*}{\partial r}$ is the radial component of the electric induction in the free space and ε_0 is the permittivity of the free space.

Requiring the solution to be even, we seek it in the form

$$
\begin{aligned}
\varphi(r,\theta) &= \sum_{m=1}^{\infty} X_m \frac{\sinh\left(m \ln \frac{r}{a}\right)}{m \cosh\left(m \ln \frac{b}{a}\right)} \cos(m\theta)\,, && (8)\\
\varphi^*(r,\theta) &= D_0 \ln \frac{r}{b} + \sum_{m=1}^{\infty} X_m \frac{\tanh\left(m \ln \frac{b}{a}\right)}{m} \left(\frac{b}{r}\right)^m \cos(m\theta)\,. && (9)
\end{aligned}
$$

These expansions satisfy the boundary conditions (4) and (5). Substitution (8) - (9) into the conditions (6) and (7) leads to the dual series equations

$$
\sum_{m=1}^{\infty} Y_m \tanh(\lambda m) \frac{[1 + M(\lambda m)]}{m} \cos(m\theta) = (1+\gamma) V_0,\ \ 0 \le \theta \le \alpha; \quad (10)
$$

$$
\sum_{m=1}^{\infty} Y_m \cos(m\theta) = D_0,\ \ \alpha < \theta \le \pi; \quad (11)
$$

where $\gamma = \left(e_{15}^2 + c_{44}^E \varepsilon_{11}\right) / c_{44}^E \varepsilon_0$, $\left(1+\gamma^{-1}\right) \ln \frac{b}{a} = \lambda$,

$$
\begin{aligned}
Y_m &= X_m \left[\gamma + \tanh\left(m \ln \frac{b}{a}\right)\right], && (12)\\
M(\lambda m) &= \frac{(\gamma+1)\tanh\left(\frac{\gamma}{1+\gamma}\lambda m\right)}{\left[\gamma + \tanh\left(\frac{\gamma}{1+\gamma}\lambda m\right)\right] \tanh(\lambda m)} - 1.
\end{aligned}
$$

We differentiate the first equations and make a substitution $\theta = \pi t / 2\mathbf{K}$, where $\mathbf{K} = \mathbf{K}(k)$ is the complete elliptic integral of the first kind and k is defined by the equations $\pi \mathbf{K}\left(\sqrt{1-k^2}\right) = \lambda \mathbf{K}(k)$. Then the equations take the form

$$\sum_{m=1}^{\infty} \tanh(\lambda m)\left[1 + M(\lambda m)\right] Y_m \sin(\mu_m t) = 0,\ \ 0 \le t \le \beta; \tag{13}$$

$$\sum_{m=1}^{\infty} Y_m \cos(\mu_m t) = D_0,\ \ \beta < t \le 2\mathbf{K}. \tag{14}$$

where $\mu_m = \pi m / 2\mathbf{K}$ and $\beta = 2\alpha \mathbf{K}/\pi$.

There is a number of methods reducing trigonometric dual series equations to equations of the second kind with completely continuous operators. In the case of interest, these regular equations have a general shortcoming: L_2-norms of their operators tend to zero as $\lambda \to 0$, so that equations are ill-conditioned for small λ and may be unsuitable for mathematical interpretation or numerical evaluation. We suggest here the method of reducing the considered dual series equations to an equivalent Fredholm integral equation of the second kind which is well-conditioned and efficiently solvable.

We introduce the functions $Pn_m\left(x \mid k^2\right) = Pn_m(x)$ defined for $0 < k < 1$ via the integral representation [2]

$$Pn_m\left(x \mid k^2\right) = \int_0^x \frac{\operatorname{cn}\frac{t}{2}\cos(\mu_m t)}{\sqrt{\operatorname{cn}^2\frac{t}{2} - \operatorname{cn}^2\frac{x}{2}}} dt \tag{15}$$

where $0 \le x \le 2\mathbf{K}$, $\mathbf{K} = \mathbf{K}(k)$ is the complete elliptic integral of the first kind, $\operatorname{cn} x = \operatorname{cn}(x \mid k)$ is the Jacobian elliptic function [3, 4]. This definition gives us the discontinuous sum

$$\sum_{m=1}^{\infty} Pn_m(x)\cos(\mu_m t) = -\mathbf{K}\left(k \operatorname{sn}\frac{x}{2}\right) + \begin{cases} 0\,, & t > x; \\ \frac{\mathbf{K}\operatorname{cn}\frac{t}{2}}{\sqrt{\operatorname{cn}^2\frac{t}{2} - \operatorname{cn}^2\frac{x}{2}}}, & t < x. \end{cases} \tag{16}$$

Other integral representations can be derived from (15) by contour integration:

$$Pn_m(x) = \cosh(\lambda m) \int_0^{2\mathbf{K}} \frac{\operatorname{dn}\frac{t}{2}\cos(\mu_m t)}{\sqrt{1 - k^2 \operatorname{sn}^2\frac{x}{2}\operatorname{sn}^2\frac{t}{2}}} dt. \tag{17}$$

$$Pn_m(x) = \coth(\lambda m) \int_x^{2\mathbf{K}} \frac{\operatorname{cn}\frac{t}{2}\sin(\mu_m t)}{\sqrt{\operatorname{cn}^2\frac{x}{2} - \operatorname{cn}^2\frac{t}{2}}} dt, \tag{18}$$

where $\lambda = \pi\mathbf{K}(k')/\mathbf{K}(k)$, $\operatorname{sn} x = \operatorname{sn}(x \mid k)$ and $\operatorname{dn} x = \operatorname{dn}(x \mid k)$ are the Jacobian elliptic functions [3].

Now we can write down the corresponding Fourier series:

$$\sum_{m=1}^{\infty} \tanh(\lambda m)\, Pn_m(x) \sin(\mu_m t) = \begin{cases} 0\,, & t < x; \\ \frac{\mathbf{K} \operatorname{cn} \frac{t}{2}}{\sqrt{\operatorname{cn}^2 \frac{t}{2} - \operatorname{cn}^2 \frac{x}{2}}}, & t > x; \end{cases} \tag{19}$$

and

$$\sum_{m=1}^{\infty} \frac{1}{\cosh(\lambda m)} Pn_m(x) \cos(\mu_m t) = -\mathbf{K}\left(k \operatorname{sn} \frac{x}{2}\right) + \frac{\mathbf{K} \operatorname{dn} \frac{t}{2}}{\sqrt{1 - k^2 \operatorname{sn}^2 \frac{x}{2} \operatorname{sn}^2 \frac{t}{2}}}. \tag{20}$$

Another representation can be found by integrating by parts:

$$\frac{\pi m \operatorname{sn} \frac{x}{2} \operatorname{dn} \frac{x}{2}}{2\mathbf{K} \operatorname{cn} \frac{x}{2}} Pn_m(x) = \frac{d}{dx} \int_0^x \frac{\operatorname{sn} \frac{t}{2} \operatorname{dn} \frac{t}{2} \sin(\mu_m t)}{\sqrt{\operatorname{cn}^2 \frac{t}{2} - \operatorname{cn}^2 \frac{x}{2}}} dt. \tag{21}$$

We will seek a solution of the dual series equations (13)-(14) of the form

$$Y_m = C \int_0^{\beta} p(s)\, \omega(s)\, Pn_m(s)\, ds + C p(\beta)\, Pn_m(\beta)\ ,\ m = 1, 2, .. \,, \tag{22}$$

where $\omega(s)$ is an unknown function, $p(s) = \sqrt{\operatorname{sn} \frac{s}{2} \operatorname{dn} \frac{s}{2} / \operatorname{cn} \frac{s}{2}}$ and C is an undetermined constant.

Substituting (22) into the first equation (13) and using the discontinuous sum (16), we get

$$\begin{aligned} \int_0^t \frac{p(x)\, \omega(x)\, dx}{\sqrt{\operatorname{cn}^2 \frac{x}{2} - \operatorname{cn}^2 \frac{t}{2}}} &= -\int_0^{\beta} \omega(s)\, U(t, s)\, ds - U(t, \beta)\,, \\ \mathbf{K} \operatorname{cn} \frac{t}{2} U(t, s) &= p(s) \sum_{m=1}^{\infty} \tanh(\lambda m)\, M(\lambda m)\, Pn_m(s) \sin(\mu_m t) \end{aligned} \tag{23}$$

We note that this equation may be considered as the Abel equation for $\omega(x)$. Upon inverting the Abel equation and exploiting the formula (21), we find

$$
\begin{aligned}
(\mathbf{I}+\mathbf{H})\omega(x) &= -H(x,\beta), \ 0 \le x \le \beta; \qquad (24)\\
\mathbf{H}\omega(x) &= \int_0^\beta \omega(s) H(x,s)\, ds,\\
H(x,s) &= \frac{p(x)\,p(s)}{2\mathbf{K}^2} \sum_{m=1}^{\infty} m \tanh(\lambda m)\, M(\lambda m)\, Pn_m(s)\, Pn_m(x), \qquad (25)
\end{aligned}
$$

We easily see from the representation (15) that $Pn_m(x)$ is a continuous function for $x \in [0, 2\mathbf{K}]$. This indicates that the equation (24) is the Fredholm integral equation of the second kind with a symmetric continuous kernel and continuous right part.

We substitute (22) into (14) and interchange the order of summation and integration. The sum (16) leads to the relation

$$
\frac{D_0}{C} = -\int_0^\beta p(s)\,\mathbf{K}\left(k \,\mathrm{sn}\, \frac{s}{2}\right) \omega(s)\, ds - p(\beta)\,\mathbf{K}\left(k \,\mathrm{sn}\, \frac{\beta}{2}\right), \qquad (26)
$$

connecting the constants D_0 and C. The constant C is established by substituting (22) into the first equation (10) and setting $t = 0$

$$
\begin{aligned}
C &= \frac{(1+\gamma)\,V_0}{\int_0^\beta [F(s)+G(s)]\,\omega(s)\,ds + F(\beta) + G(\beta)},\\
F(s) &= p(s) \sum_{m=1}^{\infty} \frac{\tanh(\lambda m)}{m} M(\lambda m)\, Pn_m(s),\\
G(s) &= p(s) \sum_{m=1}^{\infty} \frac{\tanh(\lambda m)}{m} Pn_m(s).
\end{aligned}
$$

The sum $F(s)$ converges rapidly. The slowly convergent sum $G(s)$ can be transformed into the integral by inserting the integral representation (18) and interchanging the order of operations. Utilizing the sum of the trigonometrical series [3]

$$
\sum_{m=1}^{\infty} \frac{\sin(mx)}{m} = \frac{\pi - x}{2}, \ 0 \le x \le 2\pi,
$$

we obtain

$$G(s) = \frac{\pi p(s)}{4} \int_{s}^{2\mathbf{K}} \frac{(2\mathbf{K} - t)\,\mathrm{cn}\,\frac{t}{2}}{\sqrt{\mathrm{cn}^2\,\frac{s}{2} - \mathrm{cn}^2\,\frac{t}{2}}} dt.$$

We see that the treated problem is reduced to the Fredholm integral equation of the second kind (24). Our studying this integral equation is based on the orthogonality of the functions $p(x)\,Pn_m(x)$. The starting point is the observation that the integral representation (21) may be considered as a solution of the Abel integral equation

$$\frac{m}{2\mathbf{K}} \int_{0}^{t} \frac{p^2(x)\,Pn_m(x)\,dx}{\sqrt{\mathrm{cn}^2\,\frac{x}{2} - \mathrm{cn}^2\,\frac{t}{2}}} = \frac{\sin(\mu_m t)}{\mathrm{cn}\,\frac{t}{2}}. \tag{27}$$

We multiply this relation by $\frac{1}{\mathbf{K}}\,\mathrm{cn}\,\frac{t}{2}\sin\frac{\pi m t}{2\mathbf{K}}$ and integrate with respect to t between 0 and $2\mathbf{K}$. Interchanging the order of integration coupled with (18) gives

$$\frac{m\tanh(\lambda m)}{2\mathbf{K}^2} \int_{0}^{2\mathbf{K}} p^2(x)\,Pn_m(x)\,Pn_l(x)\,dx = \delta_{ml}\ , \tag{28}$$

where δ_{ml} is the Kronecker delta.

The system $Pn_m(x)$, $m = 1, 2, ..,$ is proven to be complete [2]. The corresponding Parseval formula is

$$\begin{aligned} \frac{1}{2\mathbf{K}^2} \int_{0}^{2\mathbf{K}} p^2(x)\,f^2(x)\,dx &= \sum_{m=1}^{\infty} m\tanh(\lambda m)\,f_m^2\ , \\ f_m &= \int_{0}^{2\mathbf{K}} p^2(x)\,f(x)\,Pn_m(x)\,dx. \end{aligned}$$

We define the scalar product of the functions $f(x)$ and $g(x)$ as

$$(f(x), g(x)) = \frac{1}{2\mathbf{K}^2} \int_{0}^{\beta} p^2(x)\,f(x)\,g(x)\,dx.$$

The following estimates are obtained by means of the Parseval formula

$$\begin{aligned}
((\mathbf{I}+\mathbf{H})\,\omega,\omega) &= \sum_{m=1}^{\infty} m\tanh(\lambda m)\,[1+M(\lambda m)]\,\omega_m^2 \\
&\geq \min[1+M(\lambda m)]\,(\omega,\omega)\ , \\
\|\mathbf{I}+\mathbf{H}\| &= \sup\frac{((\mathbf{I}+\mathbf{H})\,\omega,\omega)}{(\omega,\omega)} \leq \max[1+M(\lambda m)]\,.
\end{aligned}$$

Thus operator $\mathbf{I}+\mathbf{H}$ is positive defined and hence the Fredholm integral equation has unique solution. Since $0.75 < 1 + M(u) \leq 1$ for $\gamma > 0$, $u \geq 0$, the equation is well-conditioned and can be solved by numerical methods. Convergence of the iterative methods is provided by the estimate

$$\|\mathbf{H}\| = \sup\frac{(\mathbf{H}\omega,\omega)}{(\omega,\omega)} \leq \max|M(\lambda m)| < 0.25.$$

This upper estimate rapidly decreases with growing γ and if $\gamma \geq 100$, then $\|\mathbf{H}\| < 0.0025$. In the case of actual materials, the situation is much better. For example, we have:

- for the piezoceramics PZT-4 [1]: $\gamma = 1348.1644$, $\|\mathbf{H}\| < 2.52 * 10^{-4}$,
- for the piezoceramics PZT 65/35 [5]: $\gamma = 458.188$, $\|\mathbf{H}\| < 7.41 * 10^{-4}$.

An effective solution can by derived by expanding the solution into the series with respect to the small parameter$1/\gamma$ as $\lambda \ll \gamma$. The leading asymptotic terms are

$$\begin{aligned}
D_0 &= -\frac{\gamma V_0}{G_0(\alpha)}\mathbf{K}\left(k\,\mathrm{sn}\,\frac{\mathbf{K}\alpha}{\pi}\right), \\
X_m &= \frac{1}{\gamma}Y_m = \frac{V_0}{G_0(\alpha)}Pn_m\left(\frac{2\mathbf{K}\alpha}{\pi}\right), \\
G_0(\alpha) &= \frac{\mathbf{K}^2}{\pi}\int_{\alpha}^{\pi}\frac{(\pi-t)\,\mathrm{cn}\,\frac{\mathbf{K}t}{\pi}}{\sqrt{\mathrm{cn}^2\,\frac{\mathbf{K}\alpha}{\pi}-\mathrm{cn}^2\,\frac{\mathbf{K}t}{\pi}}}dt.
\end{aligned}$$

The asymptotic form of the electric potential in free space is

$$\varphi^*(r,\theta) = D_0\ln\frac{r}{b} + \frac{V_0}{G_0(\alpha)}\sum_{m=1}^{\infty}\frac{\tanh(m\lambda)}{m}\left(\frac{b}{r}\right)^m Pn_m\left(\frac{2\mathbf{K}\alpha}{\pi}\right)\cos(m\theta)\,.$$

Substituting the integral representation (18), we obtain after interchanging the order of summation and integration

$$\varphi^*(r,\theta) = \frac{V_0\mathbf{K}}{\pi G_0(\alpha)}\int_{\alpha}^{\boldsymbol{\pi}}\frac{\left[h\left(\theta-t,\frac{b}{r}\right)+h\left(\theta+t,\frac{b}{r}\right)\right]\mathrm{cn}\,\frac{\mathbf{K}t}{\pi}}{\sqrt{\mathrm{cn}^2\,\frac{\mathbf{K}t}{\pi}-\mathrm{cn}^2\,\frac{\mathbf{K}\alpha}{\pi}}}dt + D_0\ln\frac{b}{r}, \quad (29)$$

where

$$h\left(v,u\right)=\sum_{m=1}^{\infty}\frac{u^{m}}{m}\sin\left(mv\right)=\arctan\frac{u\sin\left(v\right)}{1-u\cos\left(v\right)}.$$

The leading asymptotic term of the potential $\varphi\left(r,\theta\right)$ can be written in the integral form by substituting the integral representation (15) and utilizing the trigonometric series for $\operatorname{am}\left(u\right)=\arcsin\left(\operatorname{sn}u\right)$ [3]

$$\begin{aligned}\operatorname{am}\left(u\right) &= \frac{\pi u}{2\mathbf{K}}+2\sum_{m=1}^{\infty}\frac{\exp\left(-m\lambda\right)\sin\left(\pi m u/\mathbf{K}\right)}{m\left(1+\exp\left(-2m\lambda\right)\right)},\\ \mathbf{Re}\left(\lambda\right) &> \mathbf{Im}\left(\pi u/\mathbf{K}\right).\end{aligned}$$

Finally, we find

$$\varphi\left(r,\theta\right) = -\frac{c_{44}^{E}}{e_{15}}u_{z}\left(r,\theta\right) \tag{30}$$

$$= \frac{D_0}{\gamma\mathbf{K}}\ln\frac{r}{a}+\frac{V_0\mathbf{K}}{\pi G_0\left(\alpha\right)}\mathbf{Im}\int_{-\alpha}^{\alpha}\frac{\operatorname{am}\left[\frac{\mathbf{K}}{\pi}\left(\theta-t+i\ln\frac{r}{a}\right)\right]\operatorname{cn}\frac{\mathbf{K}t}{\pi}}{\sqrt{\operatorname{cn}^2\frac{\mathbf{K}t}{\pi}-\operatorname{cn}^2\frac{\mathbf{K}\alpha}{\pi}}}dt. \tag{31}$$

Let us write down the formulas for the charge density on electrodes. It follows from (19) that for the outer electrode

$$q\left(\theta,b\right) = D_r^{*}\left(b,\theta\right)-D_r\left(b,\theta\right)=\frac{\varepsilon_0}{b}\left[\sum_{m=1}^{\infty}Y m\cos\left(m\theta\right)-D_0\right] \tag{32}$$

$$= \frac{\varepsilon_0\,\gamma V_0\mathbf{K}\operatorname{cn}\frac{\mathbf{K}\alpha}{\pi}}{bG_0\left(\alpha\right)\sqrt{\operatorname{cn}^2\frac{\mathbf{K}\theta}{\pi}-\operatorname{cn}^2\frac{\mathbf{K}\alpha}{\pi}}},\quad -\alpha\leq\theta\leq\alpha. \tag{33}$$

The capacity of this electrode is

$$C_b=-\frac{2\pi\ \varepsilon_0}{V_0}D_0=\frac{2\pi\ \varepsilon_0\,\gamma}{G_0\left(\alpha\right)}\mathbf{K}\left(k\operatorname{sn}\frac{\mathbf{K}\alpha}{\pi}\right). \tag{34}$$

For the inner electrode, we obtain by utilizing the series (20) the following distribution of the charge density

$$\frac{a}{\varepsilon_0}q\left(\theta,a\right) = \frac{a}{\varepsilon_0}D_r\left(a,\theta\right)=\sum_{m=1}^{\infty}\frac{X_m}{\cosh\left(\lambda m\right)}\cos\left(m\theta\right)$$

$$= \frac{\gamma V_0\mathbf{K}\operatorname{dn}\frac{\mathbf{K}\theta}{\pi}}{G_0\left(\alpha\right)\sqrt{1-k^2\operatorname{sn}^2\frac{\mathbf{K}\alpha}{\pi}\operatorname{sn}^2\frac{\mathbf{K}\theta}{\pi}}}+D_0 \tag{35}$$

with the total charge equal to zero.

The expression for the component D_θ at the external pipe surface has also a very simple form

$$D_\theta\,(b,\theta) = \frac{\backepsilon_0\,\gamma V_0 \mathbf{K}\,\mathrm{sign}\,(\theta)\,\mathrm{cn}\,\frac{\mathbf{K}\theta}{\pi}}{bG_0\,(\alpha)\sqrt{\mathrm{cn}^2\,\frac{\mathbf{K}\alpha}{\pi} - \mathrm{cn}^2\,\frac{\mathbf{K}\theta}{\pi}}}\ ,\ \alpha < |\theta| \leq \pi. \tag{36}$$

In conclusion, we analyze electric and elastic fields induced by film electrodes which cover the surfaces of a piezoceramic pipe polarized in its axis direction. A novel technique for solving the arising integral equations is suggested to derive simple asymptotic formulas.

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