

On the theory of field induced relaxation in disordered system *

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Abstract

We investigate the relaxation of excitations in a strongly disordered ensemble of two-level-systems (TLS). A novel nonphonon mechanism destroying the Anderson localization in strong external fields is responsible for the relaxation. The TLS are supposed to interact via the dipole-dipole interaction that can be either of electrical or of elastic origin. An external field of low frequency and high amplitude coherently changing the TLS energy splitting is shown to control the relaxation. The case of adiabatic relaxation is considered. A concept of dressed excitation is introduced which allows us to simplify the analysis of relaxation phenomena. This concept is shown to lose meaning due to the spectral diffusion if the temperature is not low enough. In this case, the relaxation is treated in terms of initial TLS and is governed by the spectral diffusion. A crossover between these two regimes is revealed. In both cases the relaxation rates are surprisingly the same. The field and temperature dependencies of the excitation lifetime and the phase coherence time are found. The relation with the recent experimental data on ultra-low temperature relaxation in glasses is discussed and shown to support the results obtained.

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1. Recently, the field induced-relaxation of excitations at low and ultra low temperatures in disordered media, in particular, in dielectric glasses, has become a matter of interest for both theoreticians and experimentalists [1 - 4]. For a long time the standard model of noninteracting two-level-systems (TLS) [5] was a good background for the interpretation of low temperature experimental data available for glasses [6]. The spectral diffusion phenomenon became the starting point of involving the interaction between the TLS inherent in disordered materials. Following Black and Halperin [7], it has been assumed for a long time that spectral diffusion is the only manifestation of this interaction. The investigations in the last decade, both experimental and theoretical, evidence that the dipole - dipole TLS interaction is responsible for many physical "universal" properties of a disordered system (i.e., quasiindependence of their chemical composition) [8, 9]. In particular, this interaction explains the relaxation properties at ultra low temperatures, e.g., the quasilinear temperature dependencies for the inverse lifetime of elementary excitations τ_1^{-1} and dephasing rate of these excitations τ_2^{-1} [10 - 12]. In spite of the fact that involvement of the dipole - dipole interaction provides a good qualitative explanation of experimental data, an essential quantitative disagreement between the theory and experiment still remains.

The problem of the relaxation of excitations in the ensemble of interacting TLS in a strongly disordered system is closely connected with Anderson localization [13]. The localization is known to occur in the discrete problem if the transition amplitude Δ_0 coupling two nearest TLS is small in comparison with the mismatch Δ of the energy splitting at these TLS. In other words, if the parameter Δ_0/Δ is small enough, an excitation cannot hop between the sites. Let an alternating external field with the frequency ω be applied to the system. The disorder is no longer a static one since the energy level mismatch becomes time-dependent: $\tilde{\Delta} = \Delta + a \cos(\omega t)$. If the amplitude of interaction between the field and the TLS a is large enough, parameter $\tilde{\Delta}$ vanishes periodically. As a result, the parameter $\Delta_0/\tilde{\Delta}$ becomes large from time to time and the *strong coupling* between the sites takes place, i.e., *resonance* occurs as $\tilde{\Delta} \leq \Delta_0$. Therefore, the excitation can hop between the sites. According to the Landau-Zener approach, such hopping has a probability close to unity if the time evolution of $\tilde{\Delta}$ is *adiabatic*, i.e., $\Delta_0^2 \geq a\omega$. In the opposite *nonadiabatic* case, $\Delta_0^2 \leq a\omega$, the hopping probability is $\Delta_0^2/a\omega$ (see, e.g., [14]). In this paper, we will consider only the case of adiabatic relaxation.

The larger the number of sites strongly coupled to each other, the stronger is the tendency to delocalization. In the static case this leads to the requirement of a slow spatial decrease of the hopping amplitude. The delocaliza-

tion is realized if the hopping amplitude decays with a distance as $1/R^\alpha$ with $\alpha < 3$ [13]. The most interesting case $\alpha = 3$ relevant to the long-range elastic or electrical dipole-dipole interaction between the defect centers is discussed, in particular, in [15, 16, 10, 12]. It was shown [10, 12] that in an amorphous dielectric the delocalization of one-particle excitations is practically absent even within the framework of anomalous diffusion [16].

In the present paper we consider the general problem of the influence of the external alternating field on the evolution of excitations in strongly disordered media in the case $\alpha = 3$ widely discussed recently in literature [15, 16, 10, 12]. If the ratio a/ω is large enough, the propagating modes are shown to appear. The results obtained are applied to the investigation of the relaxation in TLS interacting via the dipole-dipole interaction.

On the other hand, the problem, in general, is of a special interest for the interacting TLS inherent in amorphous materials. The ratio a/ω is large in a majority of the experiments dealing with the measurement of the TLS response with applying the external alternating acoustic or electromagnetic fields (see, e.g., [6]). The results obtained manifest that the relaxation properties are vastly influenced by the field.

2. Consider an ensemble of TLS. The Schrödinger equation for a single excitation can be presented in the form ($\hbar = 1$)

$$i\dot{b}_i = e_i b_i + \sum_j U_{ij} b_j. \quad (1)$$

Here b_i is the amplitude of the excited state at the site i , e_i is the excitation energy for the corresponding TLS, and U_{ij} is the hopping amplitude between sites i and j . The energies and site locations are random.

Usually, the pseudo-spin 1/2 representation is used to describe the interacting TLS in an amorphous solid [5]. Within the framework of this formalism, b_i corresponds to the upper projection of spin S^z and (see [10, 12])

$$U_{ij} = \frac{u_{ij}}{R_{ij}^3} \frac{\Delta_{0i}\Delta_{0j}}{\varepsilon_i\varepsilon_j}, \quad \varepsilon_i = \sqrt{\Delta_{0i}^2 + \Delta_i^2}. \quad (2)$$

Here Δ_{0i} and Δ_i are the TLS tunneling amplitude and the asymmetry in energy, respectively. The subsystem of "active", or resonant TLS effectively responsible for the dynamics consists of the TLS with $\Delta_i \leq \Delta_{0i}$. For this subsystem, the factor $\frac{\Delta_{0i}\Delta_{0j}}{\varepsilon_i\varepsilon_j}$ is close to unity and can be omitted. Dispersion of the parameter u_{ij} contributes no additional randomness and without loss

of generality one can assume for simplicity $|u_{ij}| = U_0$. Then, considering only the subsystem of "active" TLS, we put

$$|U_{ij}| = U_0/R_{ij}^3 \quad (3)$$

and assume the standard uniform density distribution for the energy levels $P(\varepsilon) = P_0$. At sufficiently low temperatures a vast majority of TLS are in their ground states whereas only a few of them is excited. The excited TLS are separated by long distances, and their interaction can be ignored. Equation (1) implies this fact and describes an evolution of elementary excitation in the TLS system.

The excitation initially created at the site i can leave it for the site j if the *resonant* condition is fulfilled [16, 10, 12]

$$|e_i - e_j| \leq |U_{ij}|. \quad (4)$$

The probability to find a resonant neighbor at the distance R from the site i is $P_0 (U_0/R^3) d^3R$. The total number of sites j between two concentric spheres of radii $r_1 < R_{ij} < r_2$ centered at the site i , which obeys Eq. (4), can be estimated as (see [16, 10])

$$\begin{aligned} W(r_2, r_1) &\approx P_0 \int_{r_1}^{r_2} d^3R |U(R)| \approx \chi \ln \left(\frac{r_2}{r_1} \right), \\ \chi &= 4\pi P_0 U_0. \end{aligned} \quad (5)$$

Below we investigate the case of strong localization when the parameter $\chi \ll 1$. In particular, in amorphous solids the dimensionless parameter $P_0 U_0 < 10^{-3}$ (see [12], for recent review). Therefore, the total number of resonant sites is $W(L, r_{min}) \ll 1$ for any reasonable size of a sample L and minimum inter-site distance r_{min} . This means that, whichever the energy and position of the site are, an excitation originally created at the site i never leaves this site.

3. Consider the effect of alternating field on the energy spectrum of excitations (see [17]). Due to the field, the energy splitting Δ_i acquires the oscillating part $a_i \cos(\omega t)$. We assume that the field varies sufficiently slow and its amplitude a_i is small compared with the typical scale of the excitation energy e_i :

$$\omega \ll a_i \ll e_i \quad (6)$$

in order to treat the field as a weak perturbation. The opposite case $e_i \ll \omega$ has recently been considered in [20].

A single-particle excitation evolution is described by the Schrödinger equation (1) with an external field involved

$$i\dot{b}_i = (e_i - a_i \cos(\omega t)) b_i + \sum_j U_{ij} b_j. \quad (7)$$

This equation can be rewritten in terms of Floquet state amplitude d_{in} (see Appendix)

$$\varepsilon d_{in} = (e_i - n\omega) d_{in} + \sum_{jp} T_{in;jp} d_{jp}, \quad (8)$$

$$T_{in;jp} = U_{ij} J_{p-n} \left(\frac{a_j - a_i}{\omega} \right) \quad (9)$$

with ε being the energy eigen value of the energy. Here $J_p(a_i/\omega)$ are the Bessel functions.

Equations (8) resemble those usually considered in studying the Anderson localization when the disorder is static. If the hopping term $T_{in;jp}$ is neglected, the eigenstates of the system in the external field correspond to localized single-site excitations of a TLS coherently dressed by n "quanta" of the frequency ω . Thus, each excitation is characterized by two indices i and n , i.e., its position and the number of dressing quanta. Therefore, the problem is reduced to investigating the delocalization of these dressed excitations (DE).

Suppose that an excitation is created at the site i . In fact, this results in a creation of DE with certain double indices $i0$. To leave the state $i0$ for another DE state jn , the condition of the resonant coupling similar to Eq. (4) should be satisfied

$$|e_i - n\omega - e_j| \leq |T_{i0;jn}| = \left| U_{ij} J_n \left(\frac{a_j - a_i}{\omega} \right) \right|. \quad (10)$$

The local amplitudes a_i vary from one site to another due to fluctuations of the coupling constant. Let us denote the average of the difference $|a_i - a_j|$ by a having the same order of the magnitude as amplitudes a_i .

The argument of the Bessel function in (9) is a large parameter of the order of $a/\omega \gg 1$. If $|n| > a/\omega$, the magnitude of corresponding Bessel function is exponentially small, entailing negligible probability of the resonant coupling. The opposite condition $a > |n| \omega$ means that, in order to have a resonant coupling, the energy difference should be at least less than the amplitude a :

$$|n| < a/\omega \rightarrow |e_i - e_j| < a. \quad (11)$$

Under condition (11) and $a/\omega \gg 1$, the Bessel function in (9) can be substituted by its asymptotic value. Omitting the standard oscillating prefactor, which plays no role in a random discrete problem, one can estimate the coupling amplitude for the *DE* (3)

$$|T_{i_0;jn}| \approx \frac{U_0}{R_{ij}^3} \sqrt{\frac{\omega}{a}}. \quad (12)$$

4. Now consider the delocalization of an excitation in the system due to the alternating field within the framework of the resonant coupling concept. Two sites i and j are in the resonance when the condition (10) is satisfied at least for some $n < a/\omega$ (see Eq. (11)). Since the energy splitting between two subsequent levels is equal to the field frequency ω , the resonance always occurs if the hopping amplitude $T_{i_0;jn}$ exceeds ω and condition (11) is fulfilled. As directly follows from Eq. (12), this happens when sites i and j are separated by the distance R_{ij} obeying the condition

$$R_{ij} < R_* = (U_0/\sqrt{a\omega})^{1/3}. \quad (13)$$

In this case all the centers with $|e_i - e_j| < a$ prove to be in the resonant coupling. Taking into account that the number of such centers in unit volume is P_0a , we find the total number of resonant neighbors in the sphere of radius $R < R_*$

$$W(R, 0) = W(R_*, 0) \left(\frac{R}{R_*}\right)^3, \quad W(R_*, 0) = \frac{4\pi}{3} P_0 U_0 \sqrt{\frac{a}{\omega}}. \quad (14)$$

At the longer distances $R > R_*$, the coupling amplitude (12) is less than ω . For this reason, the probability for a genuine resonance decreases by the factor $|T_{i_0;jn}|/\omega$. As a result, the total number of resonant centers in the layer $R_* < r_1 < R < r_2$ equals to

$$\begin{aligned} W(r_1, r_2) &\approx \int_{r_1}^{r_2} d^3R (P_0a) \left(\frac{U_0}{R^3} \sqrt{\frac{a}{\omega}}\right) = \chi_* \ln \frac{r_2}{r_1} \\ \chi_* &= 4\pi P_0 U_0 \sqrt{\frac{a}{\omega}}. \end{aligned} \quad (15)$$

One should pay attention that this result differs from Eq. (5) by the factor $\sqrt{a/\omega}$.

According to [16], the parameter χ_* is a decisive parameter for the delocalization phenomena. As follows from Eqs. (14) and (15), when $\chi_* \ll 1$,

the first resonance occurs at the distance $R_1 \sim R_* e^{1/\chi_*} \gg R_*$. Thus, the first hop of the excitation occurs to the distance R_1 , taking some time t_1 determined by the inverse hopping amplitude $t_1 \sim R_1^3/U_0 \propto e^{3/\chi_*}$. Then, the time required for the second hop, when the next resonance emerges, is exponentially large compared with t_1 [16]. Therefore, the delocalization is exponentially slow, if any.

Let us consider the opposite case $\chi_* > 1$. Then, the number of resonances exceeds unity in each spherical layer $r_1 < R < 2r_1$ ($r_1 > R_*$) and the resonant sites form an *infinite cluster* implying a delocalized state [10]. Thus, when the ratio of the external field amplitude to the frequency is sufficiently high to provide the condition $\chi_* > 1$, delocalization of excitations takes place

The inverse time of a single hop between two nearest resonant neighbors in the delocalization regime can be treated as a relaxation rate for a *DE* located at certain site. At $\chi_* \gg 1$ the distance $R_1 < R_*$ between these neighbors can be estimated from the relation $W(R_1, 0) \approx 1$ (see Eq. (14)). The parameter $R_1 \approx (3/(4\pi P_0 a))^{1/3}$ and the typical hopping amplitude corresponding to this distance is given by Eq. (12)

$$T(R_1) = \frac{U_0}{R_1^3} \sqrt{\omega/a} \approx \omega \chi_*. \quad (16)$$

This expression gives an estimate of the inverse lifetime or relaxation rate for the *DE* at an arbitrary site

$$\tau_*^{-1} \approx \omega \chi_*. \quad (17)$$

The result obtained is valid for zero (very low) temperature. The point is that at non-zero temperature there exists a finite concentration of excited DE. Their relaxation with the rate τ_*^{-1} results in a spectral diffusion that can destroy the ladder structure of DE.

One can find the spectral diffusion induced transverse relaxation time τ_2 , using the relation obtained in [11]. For the dipole-dipole interaction, the rate of spectral diffusion is $P_0 U_0 T / \tau_1$. For the time t , the spectral diffusion results in the TLS energy fluctuation

$$\delta E(t) \sim (P_0 U_0) T t / \tau_1.$$

Correspondingly, the phase fluctuation is

$$\delta \phi(t) = \delta E(t) t \sim (P_0 U_0) T t^2 / \tau_1.$$

By definition, $\delta\phi(\tau_2) \sim 1$ or

$$(P_0 U_0) T \tau_2^2 / \tau_1 \approx 1. \quad (18)$$

Therefore, in the case of **adiabatic** relaxation

$$\tau_2^{-1} = a^{1/4} \omega^{1/4} T^{1/2} (P_0 U_0). \quad (19)$$

The concept of DE is valid if we are interested in the time interval t satisfying the condition

$$t < \tau_2. \quad (20)$$

The uncertainty relation for the energy $\delta E \cdot t > 1$ combined with Eq. (20) results in the condition $\delta E \cdot \tau_2 > 1$. The ladder structure of DE is not destroyed if $\delta E < \omega$ what means

$$\omega \tau_2 > 1 \quad (21)$$

or

$$\omega > a^{1/3} T^{2/3} (P_0 U_0)^{4/3}. \quad (22)$$

5. In the opposite case the ladder structure of DE is destroyed and one should return to investigating the relaxation of initial TLS. Let us make use of the Landau-Zener approach, considering the transitions due to the slow, *adiabatic*, field-induced energy-level crossing.

Let an excited site 0 be chosen. During the period of a field oscillation the crossing occurs for the sites j obeying the condition $|e_0 - e_j| < a$. Within the framework of the Landau-Zener approach, the excitation hops to the site j *with a probability close to unity* if the level e_j moving with velocity $a\omega$ passes over the energy band of the order of hopping amplitude (U_0/R_{ij}^3) for the time interval shorter than the coupling time $(U_0/R_{ij}^3)^{-1}$. The direct estimation shows that the actual transition can take place for the centers located within the sphere of radius R_* (13). The number of the neighbors obeying condition (11) appears to be equal to χ_* , and thereby, the average interval between energy levels of these χ_* centers is a/χ_* . Since the levels approach with the velocity $a\omega$, the time needed for the first effective crossing to occur, meaning that the excitation leaves the site i , is estimated just as $\frac{a}{\chi_*}/a\omega$, what surprisingly coincides with τ_* (see Eq. (17)). This is just the longitudinal relaxation time τ_1 . It is evident that dephasing rate also is given by Eq. (19)

An important note should be made. To have the **real** transition, the phase coherence should be lost for the time shorter than $t_a < \omega^{-1}$. This limitation results in the condition

$$\omega\tau_2 < 1, \quad (23)$$

which just corresponds to the condition of DE destruction, opposite to Eq. (21).

The subsystem of *resonant* TLS experiences an *adiabatic* relaxation if the average distance between TLS of the subsystem $R < R_*$ (see Eq. (13)).

The excitations can hop between two sites if their energy levels cross each other. This crossing is due to the fact that external field moves the TLS energy levels. This motion takes place within energy interval of the order of a . For this reason, the concentration of resonant TLS which could experience a relaxation (irrespective of adiabatic or nonadiabatic one!) is P_0a and, respectively, the average distance R_a between the *resonant* TLS with tunneling parameters distributed within this interval is

$$R_a \sim (P_0a)^{-1/3}. \quad (24)$$

Thus, if $R_a < R_*$, the relaxation is adiabatic, what means that

$$\omega < a(P_0U_0)^2. \quad (25)$$

For higher frequencies the relaxation is nonadiabatic.

6. Consider the ways to observe the delocalization effects in the ensemble of TLS in the amorphous solids. Several conditions should be satisfied including the delocalization condition $\chi_* > 1$ and the smallness of the external field amplitude in comparison with the characteristic energy of levels $a < T$ (see Eq. (6)).

We take the parameter $P_0U_0 \sim 4 \times 10^{-4}$ like in vitreous silica [6]. Then, for the field low frequency $\nu = \omega/2\pi \sim 100\text{Hz}$ the condition $a > 1\text{mK}$ is certainly sufficient to provide the regime $\chi_* > 1$. Thus, the restriction $T > a > 1\text{mK}$ should be fulfilled. The value $a \sim 1\text{mK}$ is physically reasonable. Indeed, in the case of acoustic external field $a = \gamma\varepsilon$ where γ is the coupling constant and ε is the strain field. Provided the typical value $\gamma \sim 10^4 K$ is taken [6], the required strain amplitude is $\varepsilon \sim 10^{-7}$. This is easy to reach in the experiments [6].

It seems that the relaxation mechanism proposed can be revealed in the internal friction experiments dealing, e.g., with the acoustic wave absorption.

The behavior of the internal friction Q^{-1} , i.e., ratio of the energy fed into the ensemble of TLS per unit time to the energy current of the sound wave, depends on the parameter $\omega\tau$ with τ^{-1} being the TLS relaxation rate.

If $\omega\tau < 1$, the internal friction gains the $\omega\tau$ -independent constant value $Q^{-1} \sim (P_0U_0)$ (plateau regime) regardless of the relaxation mechanism. According to Eq. (17), when $\chi_* > 1$ one has $\omega\tau_* < 1$. (Since this condition is valid for all centers belonging to the resonant subsystem of the TLS obeying condition (6), the parameter τ_* plays a role of macroscopic relaxation time.) Therefore, the field-induced relaxation proposed should result just in the plateau regime. The phonon-induced TLS relaxation results in the same plateau regime if the temperature is sufficiently large to provide the condition $\omega\tau_{ph} < 1$. This condition breaks down as the temperature decreases. For vitreous silica, this happens at the temperatures $T < 50\text{mK}$ if $\nu \sim 100\text{Hz}$ (see [6]). Below this temperature $\omega\tau_{ph} > 1$ and the phonon-induced internal friction decays as $Q^{-1} \sim T^3$. At the same time the field-induced relaxation being temperature-independent predominates at these temperatures when $\chi_* > 1$.

Thus the relaxation mechanism suggested should be studied within the condition $1\text{mK} < a < T < 50\text{mK}$ for frequencies $\nu \sim 100\text{Hz}$. If the amplitude a becomes smaller than 1mK , one has $\chi_* \ll 1$ and the field-induced relaxation vanishes. Keeping in mind the same frequency and temperature region, one should find a sharp decrease in Q^{-1} .

Consider the relation of the results obtained with our previous works [10] and experimental data available [18, 19]. The many-body relaxation mechanism [10] essential when the phonon relaxation is suppressed realizes without account of the strong external alternating field effect. The relaxation rate predicted in [10] is lower than the rate obtained (see Eq. (17)) in the delocalization regime $\chi_* > 1$. However, below the localization threshold $\chi_* \ll 1$ when the field-induced relaxation is suppressed but condition $a/\omega \gg 1$ continues to hold for, the field can affect the many-body relaxation. In fact, the density of resonant pairs crucial for the many-body relaxation approach developed in [10] increases by a factor $\sqrt{a/\omega}$ just like the resonance probability Eq. (15). Therefore, the alternating field can remarkably enhance the many-body relaxation even in the lack of the direct relaxation induced by the alternating field. One can think that these effects may explain the quantitative disagreement between the theoretical predictions of papers [10] and the experimental data available [18]. On the other hand, the results of paper [19], corresponding to the extremely low strain field amplitude, show no anomalies in the low temperature behavior of the internal friction. Possibly, this negative result is an indirect evidence in favor of the influence of

the strong alternating field on the relaxation rate reported in Ref. [18]. In addition, the frequency used in Ref. [19] is much larger than the frequencies used in [18] and the many-body contribution into the internal friction could be suppressed at such high frequency [12, 10].

In a number of low temperature measurements [18], the temperature-independent background has been observed in the internal friction or dielectric losses. Usually, this background is treated as a kind of experimental errors while in our opinion this can be due to the relaxation mechanism proposed. In fact, this mechanism leads to the temperature-independent relaxation rate. The study of the field amplitude and frequency dependences of the background can be useful to check such possibility.

Recently, papers [1, 2] have appeared, which seem to strongly support the results obtained. One of the main conclusions of these papers is that relaxation rate $\tau_1^{-1} \sim \sqrt{E}$ with E being the strength of the external alternating field. Nevertheless, additional experimental study of the field amplitude and frequency dependences for the relaxation rate at low temperatures, as well as a careful analysis of the experimental data interpretation are needed to shed a further light on the problem.

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Appendix

To solve Eq. (7), let us introduce the partial amplitudes of the Floquet states c_{in} so that

$$b_i = \exp[-i\varepsilon t] \sum_n c_{in} \exp(-in\omega t)$$

(see, e.g., [21]). Substituting this expression into Eq. (7) and taking the Fourier transformation of the equation obtained, we arrive at the equation

$$\varepsilon c_{im} = (e_i - m\omega) c_{im} - \frac{a_i}{2} (c_{i,m+1} + c_{i,m-1}) + \sum_j U_{ij} c_{jm}.$$

Then we multiply the last equation by the Bessel function $J_{n-m}(\frac{a_i}{\omega})$ and perform summation over m . The result can be presented in the form

$$\begin{aligned} \varepsilon \sum_m c_{im} J_{n-m}(\frac{a_i}{\omega}) &= (e_i - n\omega) \sum_m c_{im} J_{n-m}(\frac{a_i}{\omega}) + \\ &+ \sum_{jm} U_{ij} c_{jm} J_{n-m}(\frac{a_i}{\omega}) + \sum_m c_{im} ((n-m)\omega \cdot \\ &\cdot J_{n-m}(\frac{a_i}{\omega}) - \frac{a_i}{2} (J_{(n-m)+1}(\frac{a_i}{\omega}) + J_{(n-m)+1}(\frac{a_i}{\omega}))). \end{aligned} \quad (26)$$

Due to the well known properties of Bessel functions [22], each term in the last sum of the above equation vanishes. Finally, making use of the Graph summation formulae for Bessel functions [22]

$$J_{n-m}(\frac{a_i}{\omega}) = \sum_p J_{p-n}(\frac{a_j}{\omega} - \frac{a_i}{\omega}) J_{p-m}(\frac{a_j}{\omega})$$

in the sum \sum_{jm} in (26) and introducing the notation

$$d_{in} = \sum_m c_{im} J_{n-m}(\frac{a_i}{\omega}),$$

we obtain

$$\begin{aligned} \varepsilon d_{in} &= (e_i - n\omega) d_{in} + \sum_{jp} \tilde{U}_{in;jp} d_{jp}, \\ \tilde{U}_{in;jp} &= U_{ij} J_{p-n}(\frac{a_j}{\omega} - \frac{a_i}{\omega}). \end{aligned} \quad (27)$$

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