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Lifting of the Landau level degeneracy in 2D electron gas by point impurities *

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Abstract

We study the two-dimensional electron gas in magnetic field with scattering on point impurities uniformly distributed in the sample. We show that the electron-impurity interaction completely lifts the Landau level degeneracy even at low impurity concentration. This statement is in contradiction with the previous results obtained (F. Wegner, Z. Phys. B **51**, 279 (1983); E. Brezin, D.I. Gross, and C. Itzykson, Nucl. Phys. B **235**, 24 (1984); W. Apel, J. Phys. C **20**, L577 (1987); Y. Avishai, M.Ya. Azbel and S.A. Gredeskul, Phys. Rev. B **48**, 17280 (1996); S.A. Gredeskul, M. Zusman, Y. Avishai, and M.Ya. Azbel, Phys. Rep. **288**, 223 (1997)) in the unphysical approximation of two-dimensional impurities. We calculate the density of states in the self-consistent approximation taking into account multiple impurity scattering. There is a large region of the electron energy ω, measured from a Landau level, where the density of states $\rho(\omega)$ is inversely proportional to $|\omega|$ and proportional to the impurity concentration. Our results are applicable to different 2D electron systems as heterostructures, inversion layers and interfaces of condensed media.

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1. Two-dimensional (2D) electron systems are realized on interfaces of two condensed media. The typical examples of such systems are heterostructures [2], inversion layers and electrons on the helium surface [1]. In magnetic field the spectrum of 2D electrons is discrete and infinitely degenerate. We consider the removal of this degeneracy caused by interaction of electrons with point impurities. There is an opinion that at low surface concentration n_s of these impurities the infinite degeneracy the Landau levels is lifted only partially [3]. More precisely, if $n_s < \frac{1}{2\pi l_H^2}$, the Landau level degeneracy is $\frac{S}{2\pi l_H^2} - N$. Here $N = n_s S$ is the total number of impurities, S is the surface accessible for electrons, $l_H = \sqrt{\hbar c/eH}$ is the magnetic length. In this case N electron states split from each Landau level and form the impurity band. The proof of this statement [3] is based on the possibility to determine the electron wave function on a Landau level in such a way that it is zero at the points where the impurities are situated. In this case point impurities do not influence the spectrum of $\frac{S}{2\pi l_H^2} - N$ electron states. However, such a conclusion assumes the surface impurity density n_S to be determined. The problem is that 2D electron systems are usually open. The electron motion along the z axis perpendicular to the conducting planes is characterized by the wave function $\varphi(z)$ confined on the macroscopic scale z_∞ depending on the sample geometry. A typical wave function $\varphi(z)$ has the form [1, 2]

$$\varphi^2(z) = \frac{1}{2z_0} z_*^2 e^{-z_*} \quad \text{at} \quad 0 < z < z_\infty,$$
$$\varphi(z) = 0 \quad \text{at} \quad z < 0, \quad z_* \equiv z/z_0. \tag{1}$$

Electrons can move in the region $0 < z < z_\infty$ (not only in $z \le z_0$). Hence, even at low bulk concentration n_{imp} of impurities their surface density $n_S = n_{imp} z_\infty$ is very large (the effective interaction of electrons with an impurity now depends on the distance of this impurity from the conducting plane). Usually one can put $n_S = \infty$. Hence, the infinite degeneracy of the Landau levels is lifted completely by point impurities. This fact has a substantial impact on the electron transport and the quantum Hall effect. In other words, the number of impurities is small in the region $z \le z_0$ but large in the region $z < z_\infty : C_\infty = C_0 \frac{z_\infty}{z_0} \gg 1$. This situation usually takes place in experiment [1], [2]. For electrons on the liquid helium surface the heavy atoms of helium vapor play the role of impurities. Below we calculate the electron density ρ at low impurity concentration $C_0 \equiv n_{imp} 2\pi l_H^2 z_0 \ll 1$, that shows the importance of 3D impurity distribution.

2. The density of states $\rho(E)$ is related to the imaginary part of the electron Green's function $G(r, r', E)$ by the relation $\rho(E) = \frac{-1}{\pi} \mathrm{Im} G(E)$,

where $G(E) = \int d^3r G(r,r,E)$ [4]. Since the electron system is uniform in the conducting (x,y) plane, the Green's function does not depend on (x,y): $G(r,r,E) = G(z,E) = G(E)\varphi^2(z)$. The last equality implies that all electrons are on the lowest quantum size energy level and have the same wave function $\varphi^2(z)$. The magnetic field H is assumed to be strong so that the electron interaction with impurities $V(\mathbf{r})$ does not mix the electron wave functions of different Landau levels. Hence, the density of states $\rho(E)$ depends only on the quantity $\omega \equiv E - \varepsilon_0 - (n+1/2)\omega_c$, where ε_0 is the energy of size quantization along the z-axis and ω_c is the cyclotron energy. The interaction potential of electrons with point impurities is

$$V(\mathbf{r}) = u_0 \sum_i \delta(x - x_i)\delta(y - y_i)\ \delta(z - z_i). \tag{2}$$

Here x, y, z, x_i, y_i, z_i are the electron and impurity coordinates. The scattering length is equal to

$$a = -\frac{m}{2\pi\hbar^2} u_0. \tag{3}$$

The Green's function $G(\omega)$ is related to the unperturbed Green's function $G_0(\omega)$ of a pure electron system by the well-known formula [4]

$$G(\omega) = \frac{1}{G_0^{-1}(\omega) - \Sigma(\omega)}, \qquad G_0(\omega) = \frac{1}{\omega}. \tag{4}$$

The function $\Sigma(\omega)$ in (4) is [4 - 6]

$$\Sigma(\omega) = u_0 n_{imp} \int_0^{z_\infty} \frac{\varphi_0^2(z)dz}{1 - \frac{u_0}{2\pi l_H^2}\varphi_0^2(z)G(\omega)}. \tag{5}$$

This expression corresponds to the summation of all diagrams without intersection of impurity lines [6] and subsequent averaging over the impurity positions. The formulas (4),(5) are valid only for point impurities (2). It is convenient to determine the reduced wave function φ_* of perpendicular-to-layers electron motion:

$$\varphi_0^2(z) = \frac{1}{z_0}\varphi_*^2(z_*), \qquad z_* \equiv \frac{z}{z_0}, \qquad \int \varphi_*^2(z_*)\ dz_* = 1, \tag{6}$$

where z_0 is the characteristic scale of the function $\varphi_0(z)$. From (3), (5), (6) we get

$$\Sigma(\omega) = -G^{-1}(\omega)C_0 J(\omega); \qquad J(\omega) \equiv \int_0^{z_\infty/z_0} \frac{\varphi_*^2(z_*)dz_*}{\varphi_*^2(z_*) - \frac{1}{\omega_0 G(\omega)}}, \tag{7}$$

where $C_0 \equiv 2\pi l_H^2 z_0 n_{imp}$; $\omega_0 \equiv -\frac{a}{z_0}\omega_c$.

3. To begin with we restrict ourselves to the first order in the impurity concentration replacing the exact Green's function in formulas (5), (7) by the unperturbed Green's function $G_0 = 1/\omega$. This approximation means the summation of all diagrams corresponding to the scattering by a single impurity. From (4), (7) one sees that the Green's function G formally has a pole at $\omega = 0$:

$$G = \frac{1}{\omega[1 + C_0 J(\omega)]}. \tag{8}$$

However, as follows from (7), at $\omega \to 0$ the integral over z_* diverges, that leads to the small value of the residue at this pole:

$$\frac{1}{\pi}\,\mathrm{Im}G(\omega) = \delta(\omega)\frac{1}{1 + C_\infty}; \qquad C_\infty = C_0\frac{z_\infty}{z_0}. \tag{9}$$

For $z_\infty = \infty$ the residue is zero and the infinite degeneracy of Landau levels is lifted completely even in the first order in the impurity concentration. The density of states $\rho(\omega)$ is determined by the function $J(\omega)$ (7)

$$\rho(\omega) = \frac{-C_0 \mathrm{Im}J(\omega)}{\pi\omega[(1 + C_0\mathrm{Re}J(\omega))^2 + (C_0\mathrm{Im}J(\omega))^2]}. \tag{10}$$

First, we consider the model case when the reduced wave function φ_* has the form

$$\varphi_*^2(z_*) = e^{-z_*} \quad \text{at} \quad z_* > 0 \tag{11}$$
$$\varphi_*^2(z_*) = 0 \quad \text{at} \quad z_* < 0.$$

This wave function corresponds to the boundary condition at $z = 0$ for narrow and deep potential well. The function $J(\omega)$ (7) becomes

$$J(\omega) = \ln\frac{\omega - \omega_0}{\omega}. \tag{12}$$

Assuming the presence of only one type of repulsive impurities we obtain from (10), (12)

$$\rho(\omega) = \frac{C_0}{\omega\left[\left(1 + C_0 \ln\frac{\omega_0 - \omega}{\omega}\right)^2 + C_0^2\pi^2\right]}. \tag{13}$$

The density of states $\rho(\omega)$ is determined by the cut (not a pole) of the Green's function $G(\omega)$. Using the definition of ω_0 (7), we get the criterion

of the weakness of electron-impurity interaction $V(\mathbf{r})$ (3): $a \ll z_0$. Hence, if the scattering length of electron by impurity is less than the characteristic scale of the quantization along z-axis, the single Landau level approximation can be used, and the densities of states from different Landau levels do not overlap. From (13) it follows that in the wide region of ω the density of states is inversely proportional to ω:

$$\rho(\omega) = \frac{C_0}{\omega}, \quad C_0 \ln \frac{\omega_o}{\omega} \ll 1. \tag{14}$$

Up to the terms $\sim C_0^2$ the integral

$$\int \rho(\omega)\, d\omega \cong \int_0^{\omega_0} \frac{C_0 d\omega}{\omega[1 + C_0 \ln \frac{\omega_0 - \omega}{\omega}]^2} \cong 1. \tag{15}$$

Hence, the electron-impurity interaction does not change the total number of electron states but broadens the δ-peak of the unperturbed density of states. This means that at any small value of ω there is an impurity with the coordinate $z_i = z_0 \ln \frac{\omega_0}{\omega} \gg z_0$, that lifts the Landau level degeneracy. These statements do not depend on the exact shape of the wave function $\varphi(z)$. For a more realistic $\varphi(z)$ (1), the density of states $\rho(\omega)$ is given by the formula similar to (13) at $\omega \ll \omega_0$ up to the terms of the order $\ln \ln \frac{\omega_0}{\omega}$. In this case the dependence $\varphi(z) \sim e^{-z/z_0}$ at $z \gg z_0$ is essential. Moreover, there is always a wide region ω, where $\rho(\omega)$ is inversely proportional to ω. For example, for the wave function of harmonic oscillator $\varphi_*(z_*) \sim e^{-z_*^2}$, (10) and (7) at $\omega \ll \omega_0$ give:

$$\rho(\omega) = \frac{C_0}{2\omega(\ln \frac{\omega_0}{\omega})^{1/2}[1 + C_0(\ln \frac{\omega_0}{\omega})^{1/2}]^2}. \tag{16}$$

In the case of strong electric field that clamps electrons to the interface of two media: $\varphi_*^2(z_*) \sim e^{-z_*^{3/2}}$, and from (10) at $\omega \ll \omega_0$ we have

$$\rho(\omega) = \frac{2C_0}{3\omega(\ln \frac{\omega_0}{\omega})^{1/3}[1 + C_0(\ln \frac{\omega_0}{\omega})^{2/3}]^2}. \tag{17}$$

Thus, for open electron systems the electron-impurity interaction lifts completely degeneracy of the Landau levels. The density of states $\rho(\omega)$ in the first order in the impurity concentration is given by Eq. (13) - (17). The results can be easily generalized to the case of several types of impurities. For example, the analogue of (14) is

$$\rho(\omega) = \frac{C_-}{\omega} \quad \text{at} \quad \omega > 0, \tag{18}$$

$$\rho(\omega) = \frac{C_+}{|\omega|} \quad \text{at} \quad \omega < 0,$$

where C_- and C_+ are the total concentrations of repulsive and attractive impurities.

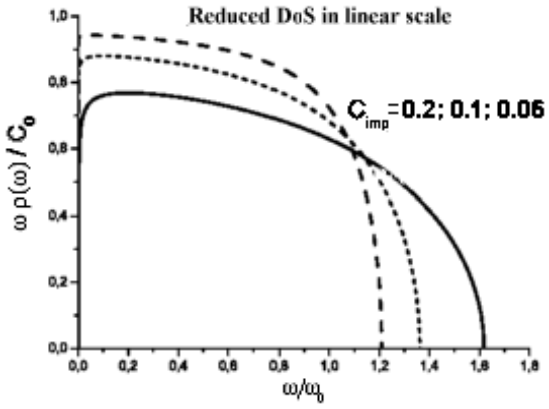

Figure 1: The reduced density of states $\omega\rho(\omega)/C_0$ as a function of the energy deviation from the Landau level for three different values of the impurity concentration: $C_0 = 0.2$ (solid line), $C_0 = 0.1$ (dot line) and $C_0 = 0.05$ (dash line). The Landau level degeneracy is completely lifted even at very low impurity concentration. One can see a wide region where $\rho(\omega) \approx C_0/\omega$.

4. The formulas (13) - (17) for $\rho(\omega)$ are valid at

$$\omega_0 e^{-1/C_0} \ll \omega, \ \omega_0 - \omega \gg e^{-1/C_0}. \tag{19}$$

Beyond this region the higher order terms in C_0 are important, due to their large logarithmic factor $\ln[(\omega_0 - \omega)/\omega]$. To study the Green's function in a wider region of ω let us consider the self-consistent approximation given by the formulas (4), (5), (7). After substitution of (11) into (7) and (4), we get an equation for G:

$$G = \frac{1}{\omega}[1 - C_0 \ln(1 - \omega_0 G)]. \tag{20}$$

At $\omega \ll \omega_0, \ \omega_0 G \gg 1$ this can be simplified:

$$G = \frac{-C_0}{\omega}[\ln(-\omega_0 G) - 1/C_0] = \frac{-C_0}{\omega} \ln\left(-\frac{e\omega_N G}{C_0}\right), \tag{21}$$

where we have introduced the new scale of energy $\omega_N \equiv \omega_0 \exp(-1/C_0)C_0/e$. After defining the new function $y \equiv -\omega G/C_0$, (21) takes the form:

$$y = \ln(y\ \omega_N/\omega) + 1. \tag{22}$$

The equation shows that y depends only on one variable: $y = y(\omega/\omega_N)$. Let us separate the real and imaginary parts of this function: $y = A + iB$. From (22) we get the system of two equations for real functions A and B, that can be simplified to:

$$A = B/\tan B$$

$$e^{B/\tan B}\frac{\sin B}{B} = \frac{\omega_N e}{\omega}. \tag{23}$$

The last equation determines the density of states $\rho(\omega) = C_0 B/\pi\omega$. We need the solution $B(\omega/\omega_N) \geq 0$. The function $B(\omega/\omega_N)$ monotonically increases from zero at $\omega = \omega_N$ to $B = \pi$ at $\omega/\omega_N \to \infty$. Near $\omega = \omega_N$ the function $B \approx \sqrt{2(\omega - \omega_N)/(\omega_N)}$. At $\omega_N \ll \omega \ll \omega_0$ the function $B = \pi[1 - 1/\ln(\omega/\omega_N)]$ up to double logarithmic terms in ω/ω_N. Thus, at $\omega \ll \omega_0$ the density of states is given by a universal function of ω/ω_N. At $C_0 \ll 1$ the asymptotics of this function are:

$$\begin{array}{lll} \omega\rho(\omega)/C_0 = \sqrt{2(\omega - \omega_N)/(\omega_N)}/\pi, & \text{at} & \omega - \omega_N \ll \omega_N; \\ \omega\rho(\omega)/C_0 = 1 - 1/\ln(\omega/\omega_N), & \text{at} & \omega \gg \omega_N. \end{array} \tag{24}$$

We consider now the region $|\omega - \omega_0| \ll \omega_0$, where in the first order in C_0 $\mathrm{Im}G_0(\omega)$ has a singularity. The solution $G(\omega)$ of the self-consistent equation (4),(5) does not have this singularity and monotonically falls to zero at $\omega = \omega_x = \omega_0(1 + x_0)$, where x_0 is the solution of algebraic equation $x_0 = C_0[\ln(1/C_0) + 1 + \ln(1 + x_0)]$. At $C_0 \ll 1$ we have

$$x_0 \approx C_0[\ln(1/C_0) + 1 + C_0 \ln(1/C_0)].$$

In the vicinity of $\omega = \omega_x$, $B(\omega) \approx \sqrt{2(\omega_x - \omega)[1/(\omega_0 C_0) - 1/\omega_x]}$. In figures 1 and 2 we plotted the reduced density of states $(\omega/C_0)\rho(\omega/\omega_0) = B/\pi$ for three different values of impurity concentration. One sees that at $\omega \ll \omega_0$ all three plots, while being shifted with ω_N, have a universal shape. The obtained solution gives the right behavior of the density of states $\rho(\omega)$ until it goes to zero, i.e. in the region $\omega_N < \omega < \omega_x$. Beyond this interval there are "tails" of the density of states that are presumably exponentially small and can be found only after taking into account the diagrams with intersection of impurity lines.

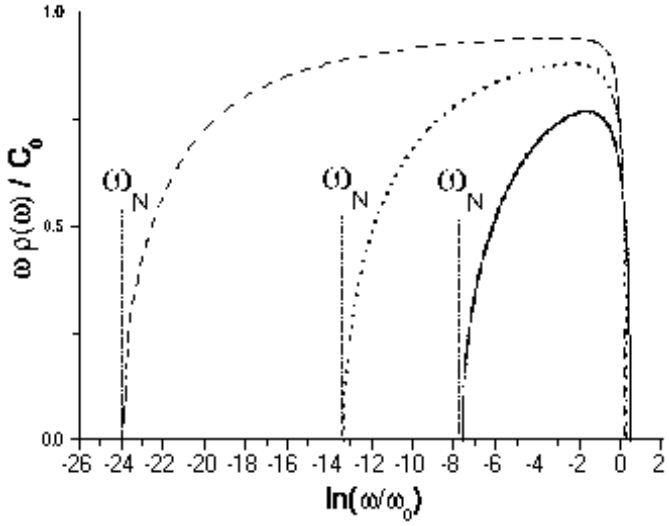

Figure 2: The reduced density of states $\omega\rho(\omega)/C_0$ in logarithmic scale for three different values of the impurity concentration: $C_0 = 0.2$ (solid line), $C_0 = 0.1$ (dot line) and $C_0 = 0.05$ (dash line). For each curve the value ω_N is marked. Near $\omega = \omega_N$ the reduced density of states is a universal function of ω/ω_N.

We note that the proposed model disagrees with the existing conception. Some exact results of the theory of electron-impurity interaction in 2D electron systems are obtained in [8, 9, 10, 11] (for a review see [12]). However, in these works two-dimensional point impurities were considered, and the interaction potential $V(r)$ (2) had the form

$$V(r) = u_0\delta(x - x_i)\delta(y - y_i). \tag{25}$$

Since the z-coordinate does not enter here, this potential corresponds to δ-shaped wires, rather than point impurity while our analysis considers three-dimensional point impurities. If one does not integrate over z-coordinate with the impurity distribution function and the wave function of electrons, one comes to the unphysical limit of δ-shaped wires.

We do not consider the exitation of electron levels in the interlayer direction, since the energy separation between these levels is $\sim 0.1eV$ which is much greater than all other energies in the problem ($\hbar\omega_c \sim 0.001eV \gg \omega_0$). The novelty of our approach consists in considering distant impurities and

demonstrating their essential role in the density of states. The electrons in our approach remain effectively two-dimensional, and the model is mathematically equivalent to that of 2D impurities with the strength distribution $\propto \varphi^2(z)$.

Very recently some exact solution [13] has been proposed for the same model with 3D impurity distribution. This exact solution has been obtained as a particular example of more general solution (Eq. (44) of [9]) derived for an arbitrary distribution of impurity strength. The result of [13] is in a qualitative agreement with our result. Thus, the dependence $\rho(\omega) = C_0/\omega$ in the main term at $\omega_0 e^{-1/C_0} \ll \omega,\ \omega_0 - \omega \gg e^{-1/C_0}$ has been proved. However, the exact solution works only for the lowest Landau level, while our perturbative solution is valid for an arbitary Landau level. Besides, no analitical formula for the whole region of parameters is obtained in [13] and in the expansion in respect with the impurity concentration (Eq. (30) of [13]) the term quadratic in concentration is missed. One can show from the high-energy expansion of the Green's function that this term must exist. A more detailed comparison of two different approaches (nonperturbative solution of Brezin et al. [9] and the perturbation theory) to the problem of 2D electron gas in magnetic field interacting with point impurities would be interesting.

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