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Performance of adaptive frequency scheduling in a group of radio communication links

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Abstract

The article treats the algorithm, potential and real efficiency of adaptive frequency scheduling procedure on instant data in group of radiolinks. The algorithmization problem is reduced to a well-known transport problem of linear programming. The figure-of-merit of performance is introduced and its potential bounds are determined. The methodology of how to take into consideration the influence of real factors on deterioration of efficiency is proposed. An example with delay of communications is presented.

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1 Introduction

In multi-user radio communication networks and izolated radiolinks it is desirable to maintain communication quality above a chosen minimum. The quality is quantified by some parameter h (as a rule, it is the carrier-to-interference ratio, CIR) and this would require that the h value in a radio link from the network be maintained above the prescribed minimal level h_0 by the scheduling of frequency channels (in the works on cellular communication systems and networks one prefers to use the term "frequency allocation"). Thus, all the scheduling methods have to use some information on h values for each treated user (link). This information is usually obtained either by the preliminary computations (based on geometry of the network's structure and properties of media of radio waves propagation) or by measurement of the CIR values for each user. Some of the works on channels assignment relevant to this problem can be found in [1, 2] and in the survey [3].

At present time it is commonly used that if a frequency channel has been assigned to a user, the channel is attached for all times of the user's communication session and cannot be changed for another one. Along with this, the feature of radio channels consists of random variety of their properties with time. Especially it is characteristically for very large distributed networks of HF band (see, for example, [4]). Thus it may be found at some moment of time that the performance quality would be far from its value at the moment of allocation. It means that if we follow some criterion of optimality at the moment of allocation, in a general case the performance will not be optimal at arbitrary instant. In this work we depart from the assumption that the user frequency must be constant for the all time of user's communication session. Consequently the network can retain the optimality at any instant. It will allow investigating the potential bounds of performance quality for each user and for the network as a whole. Along with this, these bounds are achievable only theoretically since it requires ideal, perfectly accurate information on the quality parameters h (for each user, each frequency, and at each instant). Therefore it is of practical interest to investigate also the real efficiency, which shows the influence of real factors on the performance.

The first objective of our article is to find the potential efficiency. We introduce both the criterion of optimality and the corresponding extremal problem, which solution must give the algorithm of optimal frequency scheduling. This group of problems is traditional for operation research. Then we investigate the efficiency of the obtained algorithm, understanding by this the efficiency of the system controlled by the algorithm. The corresponding results must give the potentiality. And in the final parts we treat the influence of real factors on the efficiency reducing. That class of problems is close to issues of communication theory. An example of inaccuracy due to a communication delay is considered.

2 System descriptions and scheduling

Let a group of M radiolinks operate at a common pool of N frequencies. The group members can use for their information transmission any, but only one, frequency from the pool at each instant of time. Quality communication of m-th link when it works at n-th frequency is determined by the quality parameters $h_{mn}(t)$, $m = \overline{1, M}$, $n = \overline{1, N}$, which vary randomly with time. We suppose that processes $h_{mn}(t)$ are statistically independent, that may be explained by spatial diversity of the links. The group has a control center, disposing full information about current state of the pool, that is the matrix $||h_{mn}(t)||$. The center must uninterruptedly solve the scheduling problem and allocate the frequencies to the users. The presented model by natural way can be embedded in both a cellular communication system [3] and a large distributed network [4]. Also it is suitable for a group of frequency adaptive radiolines having to perform at common pool of frequencies [5].

As it commonly accepted in assignment problems, we introduce indicator variables $a_{mn}(t) \in \{0, 1\}$, such that $a_{mn} = 1$ if at the moment t m-th user occupies n-th frequency, and $a_{mn} = 0$ otherwise. Since for each user it is sufficient to maintain the communication quality above a chosen minimum, the utility function on the set values of quality parameter h is $\nu_{mn}(t) = u[h_{mn}(t) - h_0]$, where $u(\cdot)$ is the unit-step function. Then the number of users with suitable quality is:

$$S(t) = \sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn}(t)\nu_{mn}(t) = \sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn}(t)u\left[h_{mn}(t) - h_0\right]. \quad (1)$$

Since the natural aim of the center is to provide a satisfactory performance for the maximal possible number of the group's member, we can formulate the problem of optimal scheduling as follows:

$$S(t) \Rightarrow \max_{\{a_{mn}(t)\}}, \quad (2)$$

with the conditions

$$a_{mn}(t) \in \{0, 1\}, \forall m, n; \quad \sum_{n=1}^{N} a_{mn}(t) = 1, \forall m; \quad \sum_{m=1}^{M} a_{mn}(t) \leq 1, \forall n. \quad (3)$$

The last of these conditions means that in this work we do not touch the frequency reuse problem. The problem is very actual and we intend to treat it in a separate article. Thus we confine ourselves only to the intracell and large network scheduling and also to the group operation of independent frequency adaptive links with centralized control.

The optimization problem (2)-(3) (with (1)) is the "transport problem of linear programming" with well known algorithms of its solution [6]. Thus we have reduced our problem to the well known one, that will allow us not to concern ourselves with algorithmic aspect of the issue and to pass to the efficiency analysis.

3 Potential efficiency of the scheduling

We first note at all that the algorithm of scheduling giving the allocation vector $\vec{a} = \{a_{mn_i}\}$ is a perfectly deterministic procedure as far as it operates with the *realizations* of random values h_{mn}, that is with numbers. At the same time we can consider the result $S_{\max}(t)$ of the problem solution as a random variable (process) since it is a deterministic function of random variables perfectly characterized by its distribution function.

Preliminary to consideration of the statistical characteristics, we will show that if the algorithm of assignment is optimal relatively (1)-(3), it provides maximal value of the probability that a user has a suitable communication quality under the condition of ergodicity of processes $h_{mn}(t)$. Thus, the algorithm is optimal also in that sense.

If $S_{\max}$ is a solution of the optimization problem (1)-(3), the conditional probability (under the given $S_{\max}$) that m-th user is functioning with suitable quality, is equal to $S_{\max}/M$, so the unconditional probability of this event is:

$$P = Pr(h_m \geq h_0) = \frac{1}{M} \sum_{l=1}^{M} l P_{S_m}(l) = \frac{E(S_{\max})}{M}, \tag{4}$$

where $P_{S_m}(l) = Pr(S_{\max} = l)$ is the distribution of random variable $S_{\max}$.

In consequence of the ergodicity, instead of (4), we can write with the usage of (1):

$$P = \frac{1}{M} \lim_{T \to \infty} \frac{1}{T} \int\limits_{0}^{T} S_{\max}(t)\, dt = \lim_{T \to \infty} \frac{1}{MT} \int\limits_{0}^{T} \sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn}(t) \nu_{mn}(t)\, dt. \tag{5}$$

Since the scheduling procedure maximizes the integrand in (5) at any instant of time, it provides a maximum to the parameter P.

Turn now to the evaluation of distribution function $P_{S_m}(l) = Pr(S_{\max} = l)$ of random variable $S_{\max}$. For this, note that, according to (1), it de-

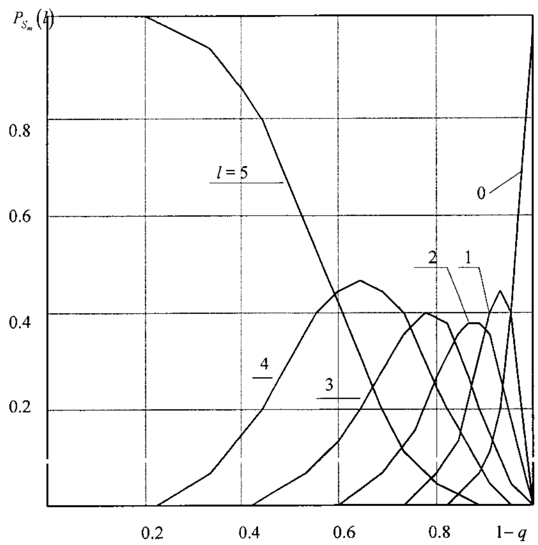

Figure 1: Probability of number of users with suitable communication versus quality of channels

pends on binary random variables ν_{mn}. Let these variables be statistically homogeneous, that is to have the same statistical properties. Then the only parameter determining their one-dimensional statistical properties, is $q = Pr(\nu = 1)$ and consequently $Pr(\nu = 0) = 1 - q$. Thus both the sought distribution $P_{S_m}(l)$ and probability P from (4) must depend only on q. Therefore hereafter we will write $P(q)$ instead of P. Note that if $f_h(h)$ is the density function (DF) of random variable h_{mn}, then $q = \int\limits_{t_0}^{\infty} f_h(h)\, dh = 1 - F_h(h_0)$, where $F_h(\cdot)$ is the distribution function of random variables h_{mn}.

The expressions for $P_{S_m}(l)$ are presented in the Appendix. However, if $l \geq 2$, the corresponding formulas become cumbersome, unsuitable for computation, and it would be more rational to use the Monte-Carlo method for evaluation of $P_{S_m}(l)$. In Fig. 1 the results of the computations are presented.

Using the values of function $P_{S_m}(l)$ in (4), we can obtain the value $P(q)$ determining the probability that the some certain user has the suitable communication quality. The results are depicted in Fig. 2. Note that for the case when single adaptive link performs at the pool of frequencies,

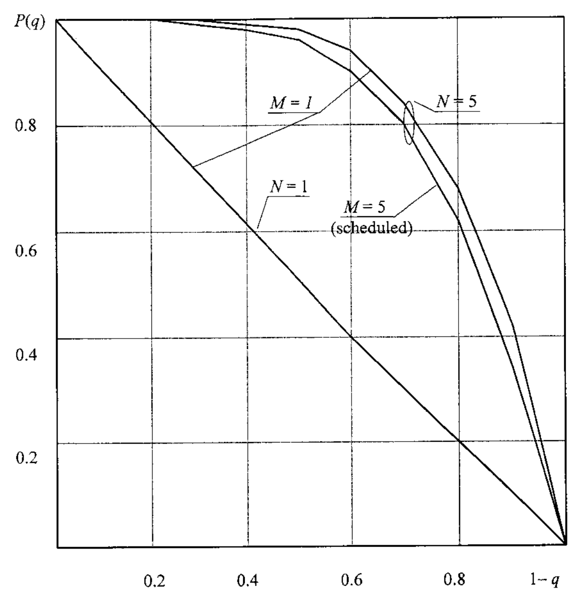

Figure 2: Probability of suitable communication quality of one channel.

$P(q) = (1-q)^N$. On the other hand, for the case of typically used performance without changes of frequency, $P(q) = 1 - q$ [6]. These graphs are also depicted in Fig. 2. From the graphs in Fig. 2 one can see that the adaptive control provides the potential performance very close to the case of a single frequency adaptive system, that is, potentially, each user nearly does not notice the presence of other users at the pool. It may be explained by statistical independence of the same frequency for the different links, which results from the assumption of spatial diversity of the links. We emphasize that, due to instant solution of the scheduling problem, the system also instantly adopts itself to the changing of its environment in such a way to have the optimal frequency allocation at the each instant, that is we deal with adaptation phenomena. Since the final result of the scheduling appears in the communication quality of each user, it would be naturally to consider $P(q)$ from (4) as a measure of the algorithm's efficiency, that is the figure-of-merit for its performance.

4 Real efficiency of the scheduling

The obtained above results relate to the case of absolutely exact control. It means that the control center possesses the perfect and plausible information about instant values of $h_{mn}(t)$, solves the scheduling problem instantly, and each link can momentarily and correctly occupy the allocated frequency. That assumption allows one to consider that the results yield the potential bound of the performance. That is why it would be of a practical interest to find out how deviations from these suppositions influence the efficiency. The state of each frequency is determined not by one but two variables $\{h_{mn}(t), z_{mn}(t)\}, m = \overline{1, M}, n = \overline{1, N}$. The first component, as before, specifies the true state of the channel but is not observable. The second component is conditionally called "estimate", it is accessible for the center and the scheduling is performed with its usage. The statistical properties of the both components are described by bivariate DF $f_{hz}(h, z)$ with the corresponding one-dimensional DF $f_h(h)$ and $f_z(z)$. The center solves the same problem (1)-(3), but instead of values $h_{mn}(t)$ it uses $z_{mn}(t)$.

Since in the course of time the frequency of m-th user can change, variable z_{mn} of m-th user at each moment coincides with the one from $z_{mn}, n = \overline{1, N}$. Consequently DF $f_{zm}(z)$ of process $z_m(t)$ differs from $f_z(z)$. Let us find this DF.

Let $q_1 = Pr(z_{mn} \geq h_o) = 1 - F_z(h_0)$ with $F_z(\cdot)$ be the distribution of process $z_{mn}(t)$ and $P(q_1)$ the probability, that m-th user gets the channel, which the center perceived as suitable. Then

$$f_{zm}(z) = P(q_1) f_z(z|z \geq h_0) + [1 - P(q_1)]\, f_z(z|z < h_0), \tag{6}$$

where $f_z(z|\cdot)$ is the conditional DF of z.

Since

$$\begin{cases} f_z(z|z \geq h_0) = \dfrac{u(z - h_0)}{q_1} f_z(z), \\ f_z(z|z < h_0) = \dfrac{u(h_0 - z)}{1 - q_1} f_z(z), \end{cases} \tag{7}$$

by substitution of (7) into (6) we obtain

$$f_{zm} = f_z(z) \left(\frac{P(q_1) u(z - h_0)}{q_1} + \frac{[1 - P(q_1)]\, u(h_0 - z)}{1 - q_1} \right). \tag{8}$$

The expression for function $P(q_1)$ is given by formula (4) if one takes into account that the value $S_{\max}$ must now be obtained by solution of the problem (1)-(3) with the usage of q_1 instead of q. The graph for $P(q_1)$ coincides with the graph for $P(q)$ with the replacing q by q_1.

The unconditional DF for the first component h_m of m-th user is

$$\begin{aligned} f_{hm}(h) &= \int_0^\infty \frac{f_{hz}(h,z)}{f_z(z)} f_{zm}(z)\, dz \\ &= \frac{1-P(q_1)}{1-q_1} \int_0^{h_0} f_{hz}(h,z)\, dz + \frac{P(q_1)}{q_1} \int_{h_0}^\infty f_{hz}(h,z)\, dz \\ &= \frac{P(q_1)}{q_1} f_h(h) + \frac{q_1 - p(q_1)}{1-q_1} \int_0^{h_0} f_{hz}(h,z)\, dz. \end{aligned} \tag{9}$$

Note that function $P(q_1)$ in this case is not the figure-of-merit of performance but the auxiliary parameter needed only for usage in (9). The demanded value quantifying the efficiency, is

$$P_r = Pr(h_{hm} \geq h_0) = \int_{h_0}^\infty f_{hm}(h)\, dh. \tag{10}$$

The last connections are of the general form which can be used for performance investigation with scheduling on inaccurate data of any kind when the decision is made with the use of estimates z_{mn} (inaccurate) but the performance is determined by true states h_{mn}. Naturally, for each case functions $f_{hz}(h,z)$ and $f_h(h)$ must be specified which is illustrated by an example below.

Scheduling in fading channels with communication delays

Fading in mobile cellular system may be divided into two different types, fast and slow fading. Fast fading is caused by multipath propagation. The other type of fading, slow, or shadow fading, is caused by obstacles in the propagation path between the mobile and the base. Slow fading has been characterized in the literature by a log-normal distribution [7].

Let the links transmit information over the channels with log-normal slow fading. For the time of performance, each link estimates the CIR values and averages then over a short term (fast) fading to obtain z. The results are transmitted to the center to use them for solution of the scheduling problem and performing the frequency allocation. Thus we deal with adaptation to slow fading. By delay we will mean the difference τ between the moment of drawing of demanded information from the channel, and the moment when the allocated channel will be occupied in fact. The value τ covers transmission of the information to the center, solution of the scheduling problem, command transmission backward to the user and execution of the command, that is carrying out the corresponding protocol.

Let the estimation be performed at the moment $t = 0$. Due to delay, the measured value $h(0)$ is realized in the form occupying an allocating channel only at the moment $t = \tau$. Then $h(0)$ can be considered as an "estimate" of the true value $h(\tau)$, which determines performance at the instant $t = \tau$. By introducing new variables $z = 10 \lg h(0)$, $h = 10 \lg h(\tau)$, we actually measure the variables in dB scale. Then the taken log-normal model of slow fading allows us to represent the two-dimensional DF of these random variables as bivariate normal DF:

$$\begin{aligned} f_{hz}(h,z) =& \frac{1}{2\pi\sigma^2\sqrt{1-\rho_\tau^2}} \exp\left\{-\frac{(z-\overline{h})^2 - 2\rho_\tau(z-\overline{h})(h-\overline{h}) + (h-\overline{h})^2}{2\sigma^2(1-\rho_\tau^2)}\right\}, \\ f_h(h) =& \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(h-\overline{h})^2}{2\sigma^2}\right\}, \end{aligned} \tag{11}$$

where $\overline{h}$ is the average value of process $10 \lg h(t)$, σ and $\rho_\tau = \rho(\tau)$ are its variance and autocorrelation function, respectively.

Substitution of (11) into (9) and then into (10), yields after some transformations the following expression for the figure-of-merit

$$P_r = 1 - \frac{P(q_1)(1-q_1)}{q_1} - \frac{q_1 - P(q_1)}{q_1(1-q_1)} \Phi_h \left(\frac{10 \lg h_0 - \overline{h}}{\sigma}, \frac{10 \lg h_0 - \overline{h}}{\sigma} \right), \tag{12}$$

with $q_1 = 1 - \Phi_h\left((10 \lg h_0 - \overline{h})/\sigma\right)$, where $\Phi_h(\cdot)$ and $\Phi_h(\cdot,\cdot)$ are one- and two-dimensional normal distribution functions [8].

The results of computation by formula (12) are represented in Fig 3.

We can also comment the results from the another position. Consider the case when the information about the channel state can be obtained only at discrete moments of time. It correspondents to the situation with periodical radiation of sounding (testing) signals at the moment $t = 0$ and determination of the channel's states resulting from these signals reception. Then we can consider that value is the time which passed after the sounding session, and Fig. 3 shows in what measure the performance deteriorates with the increasing of τ, and the information drawn from the sounding is becoming obsolete.

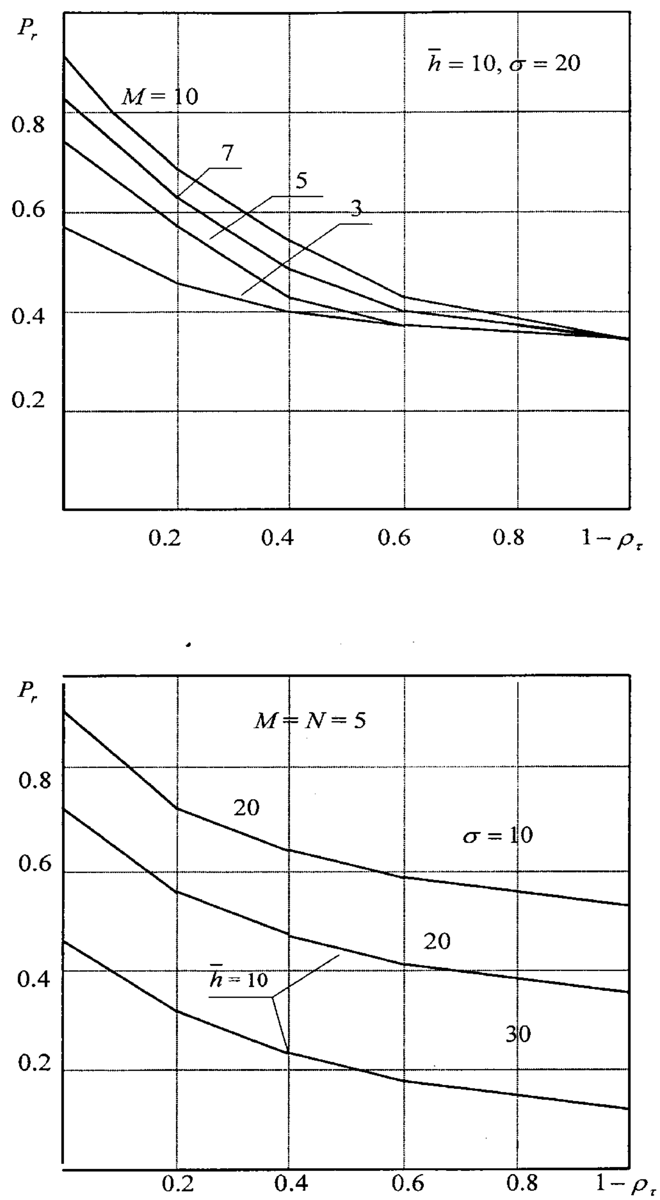

Figure 3: Dependence of communication quality for one user on delay ($M = N$).

5 Conclusion

1. The adaptive optimal frequency scheduling procedure is proposed and the potential efficiency of the procedure is found.
2. The efficiency of the optimal procedure is close to the efficiency of a single frequency adaptive link operating at the same pool of frequencies. The more is the number of users at the pool, the better is the quality of communication for each user.
3. The methodology of how to take into consideration the influence of real factors on efficiency deterioration (real efficiency), is proposed. The methodology is of rather general form and may be specified for a wide spectrum of real factors.
4. The methodology was specified for the case of delay between the moment of drawing of demanded information from the channel, and the moment when the allocated channel is occupied in fact.
5. Correlation between the moments of made decision execution and drawing of demanded information from the channels hardly affects deterioration of efficiency relative its potential value.

Our finding can be useful for practical estimation of upper performance bounds in communication networks and also for determination of real factors affecting the performance of communication networks and large distributed communication systems.

Appendix

Evaluation of distribution function $P_{S_m}(l)$

Let us assume the all processes $h_{mn}(t)$ are statistically independent. Let i_m be the number of frequencies suitable for m-th user at the instant t. Then it is evident that

$$Pr\,(i_m = k) = \binom{N}{k} q^k (1-q)^{N-k}. \tag{13}$$

Since the probability does not depend on m, hereinafter we will write $p(k)$ instead of $Pr\,(i_m = k)$. Then

$$P_{S_m}(0) = Pr(S_m = 0) = p^M(0). \tag{14}$$

For determination of $P_{S_m}(1)$ we note that $S_m = 1$ if:
(a) either an event A_1 occurs such that

$$A_1 = \left\{ i_m = 0; m \neq k; m, k = \overline{1, M} \right\} \wedge \{ i_k \geq 1 \}, \tag{15}$$

(b) or an event A_2 occurs such that

$$A_2 = \{i_m = 0; m = j_1, \ldots, j_l; l < M-1\} \wedge \{i_m = 1; m \neq j_1, \ldots, j_l\}. \quad (16)$$

Since $Pr\{S_{max} = 1|A_2\} = 1/N^{M-l-1}$, we can write

$$\begin{aligned} P_{S_m}(1) &= Pr(A_1) + Pr(A_2) \\ &= \binom{M}{1} p^{M-1}(0)[1-p(0)] + \sum_{l=0}^{M-2} \binom{M}{l} \frac{1}{N^{M-l-1}} p^l(0) p^{M-l}(1). \end{aligned} \quad (17)$$

With the same line of reasoning we can obtain

$$\begin{aligned} P_{S_m}(2) = &\sum_{k=2}^{M-2} \frac{\binom{M}{k}(N-1)(2^{M-k-1}-1)p^k(0)p^{M-1}(1)}{N^{M-k-1}} \\ &+ \sum_{k,r=2}^{N} \binom{M}{2} p(k)p(r)p^{M-2}(0) + + \sum_{k=3}^{M} \frac{\binom{M}{k}}{\binom{M}{2}} p^{M-k}(0)p^k(2) \\ &+ \sum_{k_1+k_2 \leq M, k_1 \neq 0, k_2 \neq 0}^{M} \frac{M\alpha(k_1,k_2)}{k_1!k_2!(M-k_1-k_2)\binom{M}{2}^{k_2-1} N_1^k} p(0)p(1)p(2) \\ &+ \sum_{k=3}^{M} \sum_{r=1}^{M-1} \frac{(M-r)\binom{M}{r}}{N^{r-1}} p(k)p^r(1)p^{M-r-1}(0), \end{aligned} \quad (18)$$

with

$$\alpha(k_1,k_2) = \begin{cases} N & \text{if} \quad k_1 = k_2 = 1; \\ N + 2^{k-1} - 2 & \text{if} \quad k_2 = 1, k_1 > 1; \\ 2^{k_1} & \text{if} \quad k_2 > 1. \end{cases} \quad (19)$$

In principle, one could obtain in the same manner the expression for $P_{S_m}(l)$ also for $l > 2$ but in this case formula (19) becomes cumbersome and unsuitable for computation. For $l \geq 3$ it would preferable to use the simulation method for evaluation of $P_{S_m}(l)$.

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