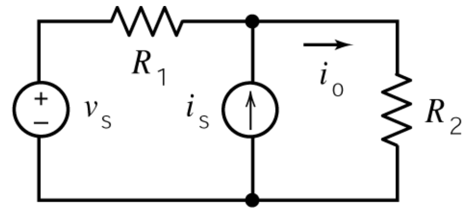
Another Sample ES 250 Final Exam



1. This circuit has two inputs, v_s and i_s, and one output i_o. The output is related to the inputs by the equation

$$i_o = a\,i_s + b\,v_s$$

Given the following two facts:

The output is i_o = 0.45 A when the inputs are i_s = 0.25 A and v_s = 15 V.

and

The output is i_o = 0.30 A when the inputs are i_s = 0.50 A and v_s = 0 V.

Determine the following:

The values of the constants a and b are a = ____0.6_____ and b = __0.02___ A/V.

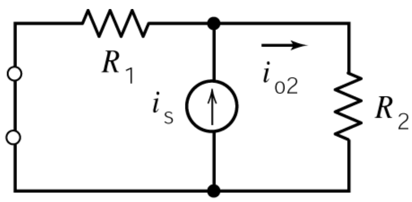
The values of the resistances are R_1 = ____30____ Ω and R_2 = ____20___ Ω.

From the 1st fact: $$0.45 = a(0.25) + b(15)$$

From the 2nd fact: $$0.30 = a(0.50) + b(0) \;\Rightarrow\; a = \frac{0.30}{0.50} = 0.60$$

Substituting gives $$0.45 = (0.60)(0.25) + b(15) \;\Rightarrow\; b = \frac{0.45 - (0.60)(0.25)}{15} = 0.02$$

Next, consider the circuit:

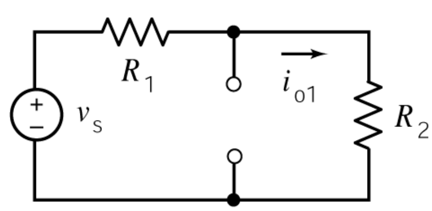


$$a\,i_s = i_{o1} = i_o\big|_{v_s=0} = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

so

$$0.60 = \frac{R_1}{R_1 + R_2} \;\Rightarrow\; 2R_1 = 3R_2$$

and



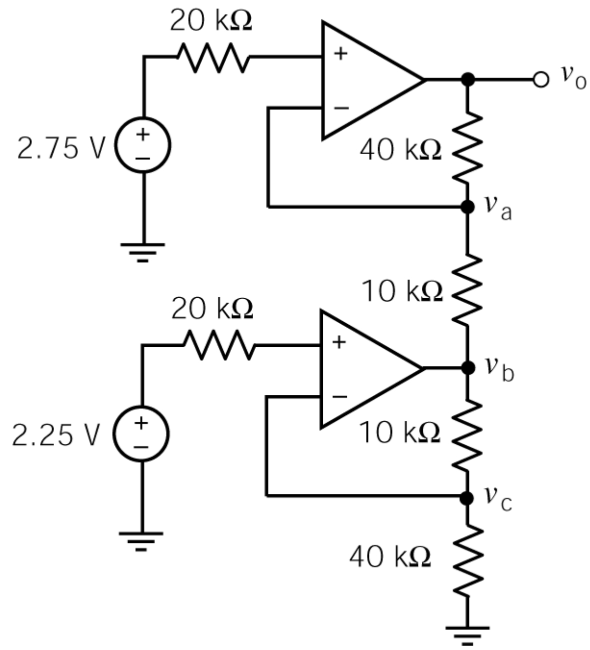
$$b\,v_s = i_{o2} = i_o\big|_{i_s=0} = \frac{v_s}{R_1 + R_2}$$

so

$$0.02 = \frac{1}{R_1 + R_2} \;\Rightarrow\; R_1 + R_2 = \frac{1}{0.02} = 50\ \Omega$$

Solving these equations gives R_1 = 30 Ω and R_2 = 20 Ω.

2. Determine the values of the node voltages v_a, v_b, v_c and v_o:



v_a = ___2.75___ V, v_b = __2.8125__ V,

v_c = ___2.25___ V, and v_o = ___2.50___ V.

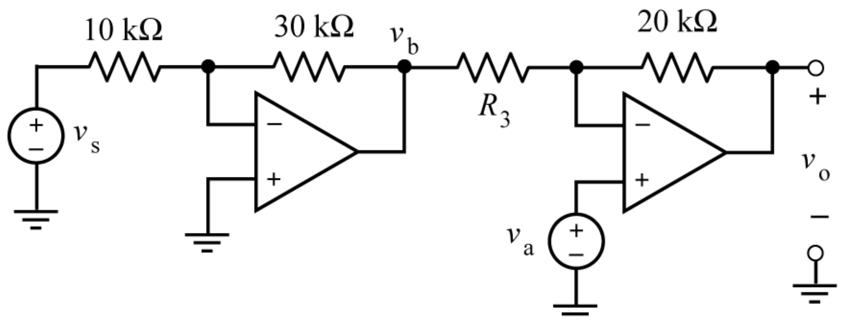
Due to the properties of the ideal op amp, $v_a = 2.75$ V and $v_c = 2.25$ V. The node equation at node c is

$$\frac{v_b - v_c}{10\times10^3} = \frac{v_c}{40\times10^3} \quad \Rightarrow \quad v_b = \frac{5}{4}v_c = 2.8125 \text{ V}$$

The node equation at node c is

$$\frac{v_o - v_a}{40\times10^3} = \frac{v_a - v_b}{10\times10^3} \quad \Rightarrow \quad v_o = 5v_a - 4v_b = 2.5 \text{ V}$$

3.



The node voltage v_b is given by

$$v_b = -3v_s$$

The input to this circuit is the voltage v_s. The output is the node voltage v_o. The output is related to the input by the equation $v_o = m\,v_s + b$ where *m* and *b* are constants.

(a) Suppose $v_o = 18$ V when $v_s = 1$ V and $v_o = 6$ V when $v_s = -1$ V. Determine the values of *m* and *b*:

m = ___6____V/V and *b* = ___12____V.

(b) Instead, suppose that R_3 = 12 kΩ and v_a = 3 V. Determine the values of *m* and *b*:

m = ___5____V/V and *b* = ___8____V.

(c) Instead, suppose that we require $v_o = 4\,v_s + 7$. Determine the required values of R_3 and v_a:

R_3 = ___15____kΩ and v_a = ___3____V.

Solution:
(a) From the given data:

$$\left.\begin{aligned}18 &= m(1)+b \\ 6 &= m(-1)+b\end{aligned}\right\} \Rightarrow 18+6 = m-m+2b \Rightarrow b = \frac{24}{2} = 12$$

Then $18 = m(1)+12 \Rightarrow m = 6$

Writing node equations at the inverting input nodes of the op amps gives:

$$\frac{v_s}{10}+\frac{v_b}{30} = 0 \Rightarrow v_b = -3v_s \quad \text{and} \quad \frac{v_b - v_a}{R_3}+\frac{v_o - v_a}{20} = 0 \Rightarrow v_o = -\frac{20}{R_3}v_b + \left(1+\frac{20}{R_3}\right)v_a$$

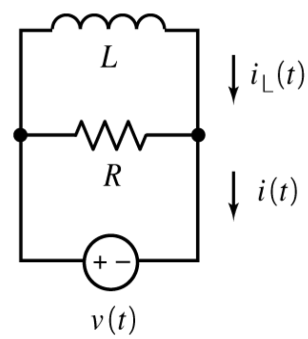
So
$$v_o = -\frac{20}{R_3}(-3v_s)+\left(1+\frac{20}{R_3}\right)v_a = \frac{60}{R_3}v_s + \left(1+\frac{20}{R_3}\right)v_a$$

(b) Substituting values gives $v_o = \frac{60}{12}v_s + \left(1+\frac{20}{12}\right)(3) = 5v_s + 8$

(c) Comparing coefficients gives

$$4 = \frac{60}{R_3} \Rightarrow R_3 = 15\ \Omega \quad \text{and} \quad 7 = \left(1+\frac{20}{R_3}\right)v_a = \left(1+\frac{20}{15}\right)v_a = \frac{7}{3}v_a \Rightarrow v_a = 3\text{ V}$$

4. The input to this circuit is the voltage:
$v(t) = 4e^{-20t}$ V for $t > 0$



The output is the current: $i(t) = -1.2e^{-20t} - 1.5$ A for $t > 0$

The initial condition is $i_L(0) = -3.5$ A . Determine the values of the resistance and inductance:

$R =$ ___5___ Ω and $L =$ ___0.1____ H.

Solution: Apply KCL at either node to get

$$i(t) = \frac{v(t)}{R} + i_L(t) = \frac{v(t)}{R} + \left[\frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0)\right]$$

That is

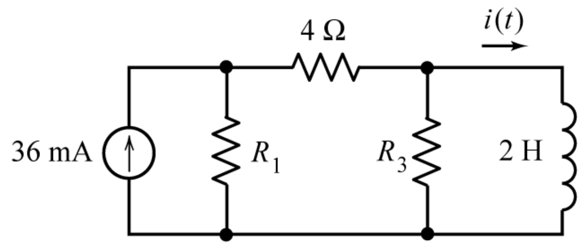
$$-1.2e^{-20t} - 1.5 = \frac{4e^{-20t}}{R} + \frac{1}{L}\int_0^t 4e^{-20\tau}\,d\tau - 3.5 = \frac{4e^{-20t}}{R} + \frac{4}{L(-20)}\left(e^{-20t}-1\right) - 3.5$$

$$= \left(\frac{4}{R} - \frac{1}{5L}\right)e^{-20t} + \frac{1}{5L} - 3.5$$

Equating coefficients gives $-1.5 = \frac{1}{5L} - 3.5 \Rightarrow L = 0.1\text{ H}$

And $-1.2 = \frac{4}{R} - \frac{1}{5L} = \frac{4}{R} - \frac{1}{5(0.1)} = \frac{4}{R} - 2 \Rightarrow R = 5\ \Omega$

5. After time $t = 0$, a given circuit is represented by this circuit diagram.



a. Suppose that the inductor current is

$$i(t) = 21.6 + 28.4e^{-4t} \text{ mA} \quad \text{for } t \geq 0$$

Determine the values of R_1 and R_3: R_1 = ___6____ Ω and R_3 = ___40____ Ω.

b. Suppose instead that R_1 = 16 Ω, R_3 = 20 Ω, the initial condition is $i(0)$ = 10 mA, and the inductor current is $i(t) = A + Be^{-at}$ for $t \geq 0$. Determine the values of the constants *A*, *B*, and *a*:

A = ___28.8__ mA, *B* = __–18.8___ mA and *a* = ____5___ s.

Solution:

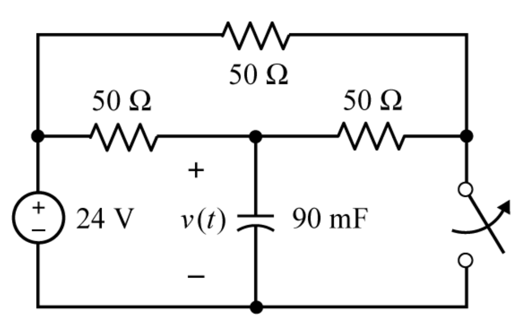
The inductor current is given by $i(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at}$ for $t \geq 0$ where $a = \frac{1}{\tau} = \frac{R_t}{L}$.

a. Comparing this to the given equation gives $21.6 = i_{sc} = \frac{R_1}{R_1 + 4}(36) \Rightarrow R_1 = 6\ \Omega$ and

$4 = \frac{R_t}{2} \Rightarrow R_t = 8\ \Omega$. Next $8 = R_t = \left(R_1 + 4\right) \| R_3 = 10 \| R_3 \Rightarrow R_3 = 40\ \Omega$.

b. $R_t = (16+4) \| 20 = 10\ \Omega$ so $a = \frac{1}{\tau} = \frac{10}{2} = 5$ s. also $i_{sc} = \frac{16}{16+4}(36) = 28.8$ mA. Then
$i(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at} = 28.8 + (10 - 28.8)e^{-5t} = 28.8 - 18.8e^{-5t}$.

6. a) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **open**:



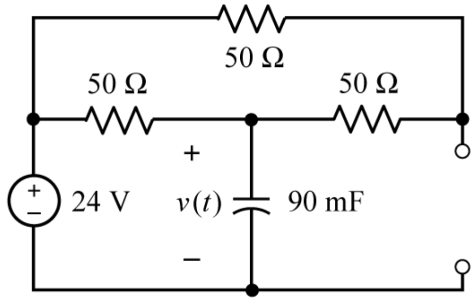
τ = _____3____ s and $v(\infty)$ = ____24____ V

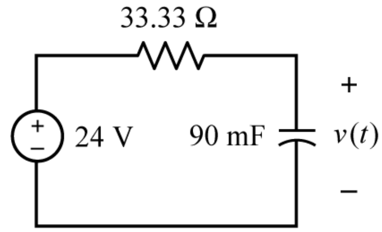
b) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **closed**:

τ = __2.25__ s and $v(\infty)$ = ____12____ V

Solution:

a.) When the switch is open we have

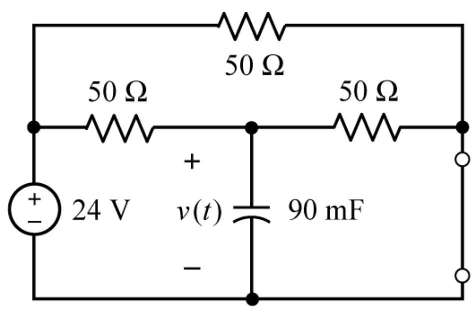




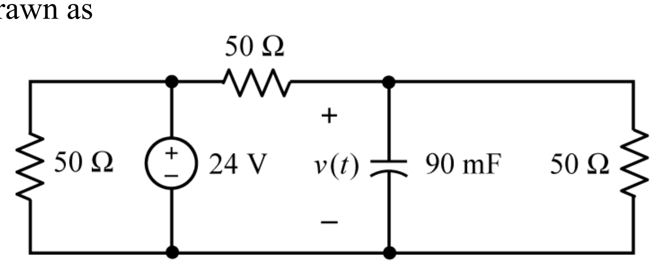
After replacing series and parallel resistors by equivalent resistors, the part of the circuit connected to the capacitor is a Thevenin equivalent circuit with $R_t = 33.33\ \Omega$. The time constant is $\tau = R_t C = 33.33(0.090) = 3$ s.

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 33.33 Ω resistor and KVL gives $v(\infty) = 24$ V.

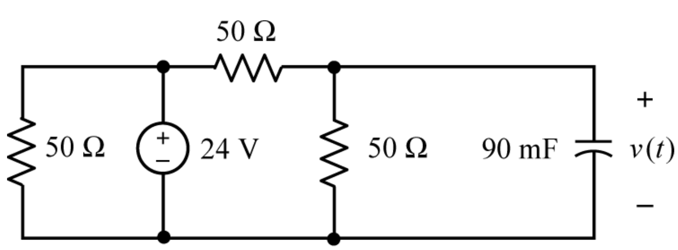
b.) When the switch is closed we have

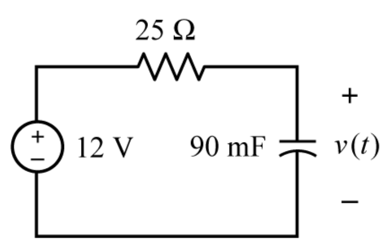


This circuit can be redrawn as



Now we find the Thevenin equivalent of the part of the circuit connected to the capacitor:



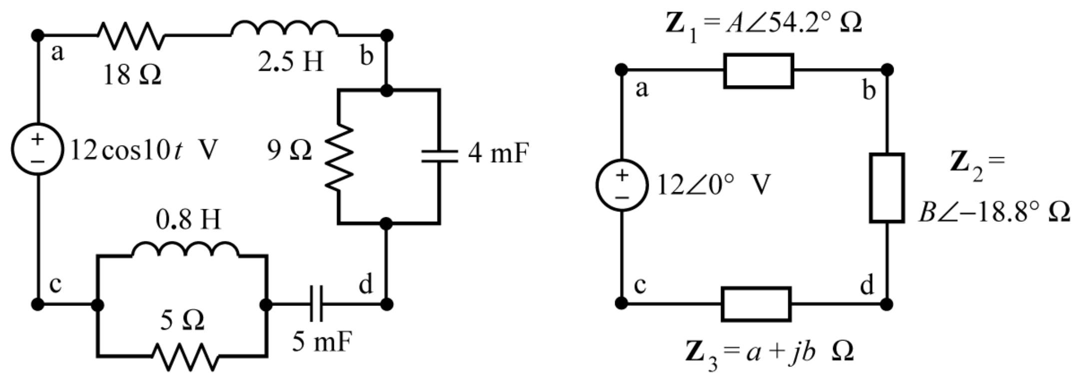


So $R_t = 25\ \Omega$ and

$$\tau = R_t C = 25(0.090) = 2.25 \text{ s}$$

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 25 Ω resistor and KVL gives $v(\infty) = 12$ V.

7. Here is an ac circuit represented in both the time domain and the frequency domain:



Determine the values of A, B, a and b.

$A =$ __30.8___ V, $B =$ __8.47__ Ω, , a = __3.57__ Ω and $b =$ __−17.75___ Ω.

Solution:
The impedance between nodes a and b is given by

$$18 + j(10)(2.5) = 18 + j25 = 30.8\angle 54.2°$$

To find the impedance between nodes b and c we first find the impedance of the capacitor:

$$-j\frac{1}{(10)(0.004)} = -j\frac{1}{0.04} = -j25$$

then

$$\frac{9(-j25)}{9 - j25} = \frac{-j225}{26.57\angle -70.2°} = \frac{225\angle -90°}{26.57\angle -70.2°} = 8.47\angle -18.8° \ \Omega$$

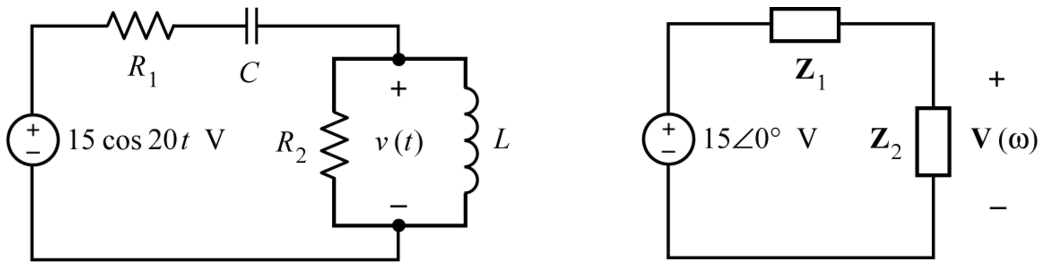
The impedance between nodes c and d is given by

$$\frac{(5)(j(10)(0.88))}{5 + j(10)(0.8)} - j\frac{1}{(10)(0.005)} = \frac{j40}{5 + j8} - j\frac{1}{0.05} = \frac{j40}{5 + j8}\left(\frac{5 - j8}{5 - j8}\right) - j20$$

$$= \frac{320 + j200}{25 + 64} - j20$$

$$= 3.60 + j2.25 - j20 = 3.60 - j17.75 \ \Omega$$

8. Here is an ac circuit represented in both the time domain and the frequency domain:



Given that $\mathbf{Z}_1 = 15.3\angle -24.1°\ \Omega$, $\mathbf{Z}_2 = 14.4\angle 36.9°\ \Omega$ and $\mathbf{V}(\omega) = A\angle 31.5°\ \text{V}$, determine the values of A, R_1, R_2, L and C.

$A =$ __8.43___ V, $R_1 =$ __14__ Ω, $R_2 =$ ___24___ Ω, $L =$ __0.9__ H and $C =$ ___8___ mF.

Solution:

Consider $\mathbf{Z}_1$:

$$R_1 - j\frac{1}{20C} = 15.3\angle -24.1° = 14 - j6.25 \quad\Rightarrow\quad R_1 = 14\ \Omega \quad \text{and} \quad C = \frac{1}{20(6.25)} = 0.008\ \text{F} = 8\ \text{mF}$$

Next consider $\mathbf{Z}_2$:

$$\frac{1}{\frac{1}{R_2} + \frac{1}{j20L}} = 14.4\angle 53.1° \quad\Rightarrow\quad \frac{1}{R_2} + \frac{1}{j20L} = \frac{1}{14.4\angle 53.1°} = \frac{1}{14.4}\angle -53.1 = 0.04167 - j0.05556$$

Equating coefficients gives

$$R_2 = \frac{1}{0.04167} = 24\ \Omega \quad \text{and} \quad L = \frac{1}{20(0.05556)} = 0.9\ \text{H}$$

Next, consider the voltage divider:

$$A\angle 31.5° = \frac{14.4\angle 36.9°}{15.3\angle -24.1° + 14.4\angle 36.9°}(15\angle 0°) = \frac{(15)(14.4)\angle 36.9°}{(14 - j6.25)(11.52 + j8.64)}$$

$$= \frac{216\angle 36.9°}{25.52 + j2.39}$$

$$= \frac{216\angle 36.9°}{25.63\angle 5.4°} = 8.43\angle 31.5°\ \text{V}$$