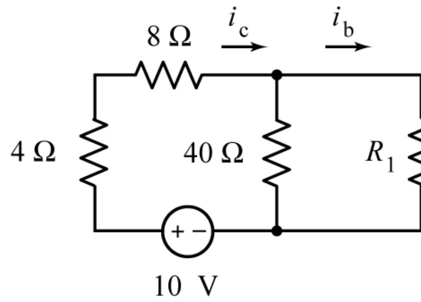
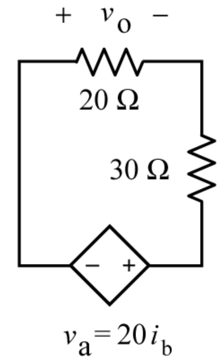
ES 250 Practice Final Exam

1. Given that

$$v_a = 8 \text{ V},$$





Determine the values of R_1 and v_o:

$R_1 =$ ____10____ Ω,

and

$v_o =$ ___−3.2___ V

First,

$$v_o = -\frac{20}{20+30}8 = -3.2 \text{ V}$$

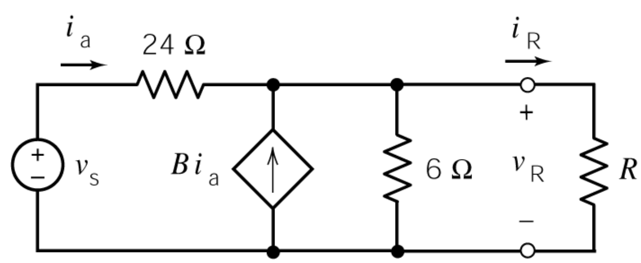
Next,

$$\frac{8}{20} = i_b = \frac{40}{40+R_1}i_c = \frac{40}{40+R_1}\left(\frac{10}{12+40 \parallel R_1}\right) = \frac{40}{40+R_1}\left(\frac{10}{12+\dfrac{40R_1}{40+R_1}}\right) = \frac{400}{12(40+R_1)+40R_1} = \frac{400}{480+52R_1}$$

Then

$$\frac{8}{20} = \frac{400}{480+52R_1} \Rightarrow 480+52R_1 = \frac{400(20)}{8} = 1000 \Rightarrow \frac{1000-480}{52} = 10 \ \Omega$$

2. Given that $0 \le R \le \infty$ in this circuit, consider these two observations:



When $R = 2\ \Omega$ then $v_R = 4$ V and $i_R = 2$ A.

When $R = 6\ \Omega$ then $v_R = 6$ V and $i_R = 1$ A.

Fill in the blanks in the following statements:

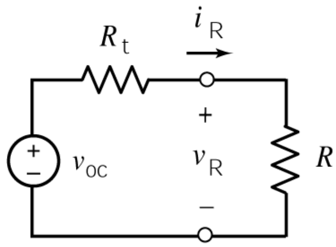
a. The maximum value of i_R is _____4_____A.

b. The maximum value of v_R is ____8_____V.

c. The maximum value of $p_R = i_R v_R$ occurs when $R =$ _____2____Ω.

d. The maximum value of $p_R = i_R v_R$ is ____8____W.

We can replace the part of the circuit to the left of the terminals by its Thevenin equivalent circuit:



Using voltage division $v_R = \dfrac{R}{R+R_t}v_{oc}$ and using Ohm's law $i_R = \dfrac{v_{oc}}{R+R_t}$.

By inspection, $v_R = \dfrac{R}{R+R_t}v_{oc} = \dfrac{v_{oc}}{1+\dfrac{R_t}{R}}$ will be maximum when $R = \infty$. The maximum value of v_R will be v_{oc}. Similarly, $i_R = \dfrac{v_{oc}}{R+R_t}$ will be maximum when $R = 0$. The maximum value of i_R will be $\frac{v_{oc}}{R_t} = i_{sc}$.

The maximum power transfer theorem tells use that $p_R = i_R v_R$ will be maximum when $R = R_t$. Then

$$p_R = i_R v_R = \left(\frac{v_{oc}}{R+R_t}\right)\left(\frac{R}{R+R_t} v_{oc}\right) = R\left(\frac{v_{oc}}{R+R_t}\right)^2.$$

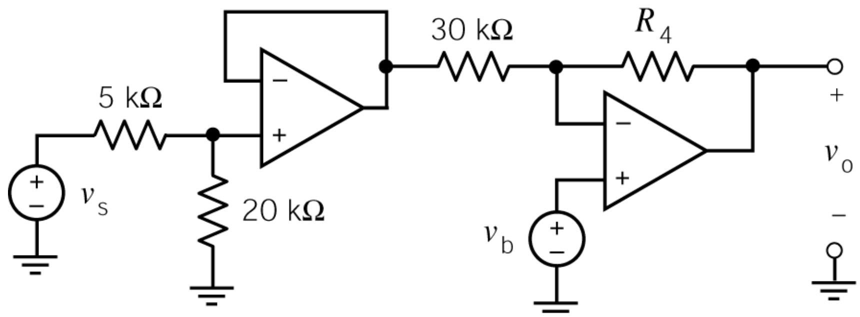
Let's substitute the given data into the equation $i_R = \frac{v_{oc}}{R+R_t}$.

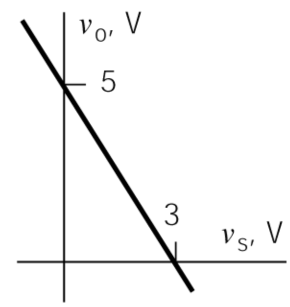
When $R = 2\ \Omega$ we get $2 = \frac{v_{oc}}{2+R_t} \Rightarrow 4+2R_t = v_{oc}$. When $R = 6\ \Omega$ we get $1 = \frac{v_{oc}}{6+R_t} \Rightarrow 6+R_t = v_{oc}$.

So $6+R_t = 4+2R_t \Rightarrow R_t = 2\ \Omega$ and $v_{oc} = 4+2R_t = 8\ \text{V}$. Also $i_{sc} = \frac{v_{oc}}{R_t} = \frac{8}{2} = 4\ \text{A}$.

Now the blanks can be easily filled-in.

3.





The input to this circuit is the voltage, v_s. The output is the voltage v_o. The voltage v_b is used to adjust the relationship between the input and output. Determine values of R_4 and v_b that cause the circuit input and output have the relationship specified by the graph

$v_b =$ ___1.62__ V and $R_4 =$ ___62.5___ kΩ.

Recognize the voltage divider, voltage follower and noninverting amplifier to write

$$v_o = \left(\frac{20\times10^3}{20\times10^3+5\times10^3}\right)\left(-\frac{R_4}{30\times10^3}\right)v_s + \left(1+\frac{R_4}{30\times10^3}\right)v_b = \left(-\frac{2R_4}{75\times10^3}\right)v_s + \left(1+\frac{R_4}{30\times10^3}\right)v_b$$

(Alternately, this equation can be obtained by writing two node equations: one at the noninverting node of the left op amp and the other at the inverting node of the right op amp.)

The equation of the straight line is $\quad v_o = -\frac{5}{3}v_s + 5$

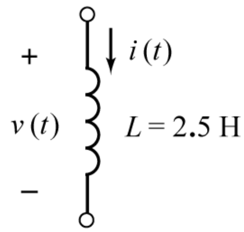
Comparing coefficients gives

$$-\frac{2R_4}{75\times10^3} = -\frac{5}{3} \quad\Rightarrow\quad R_4 = \frac{5}{3}\times\frac{75\times10^3}{2} = 62.5\times10^3 = 62.5\ \text{k}\Omega$$

and

$$5 = \left(1 + \frac{R_4}{30\times10^3}\right)v_b = \left(1 + \frac{62.5\times10^3}{30\times10^3}\right)v_b = 3.08333\, v_b \quad \Rightarrow \quad v_b = \frac{5}{3.08333} = 1.62 \text{ V}$$

4. Consider this inductor. The current and voltage are given by



$$i(t) = \begin{cases} 5t - 4.6 & 0 \le t \le 0.2 \\ a\,t + b & 0.2 \le t \le 0.5 \\ c & t \ge 0.5 \end{cases} \quad \text{and} \quad v(t) = \begin{cases} 12.5 & 0 < t < 0.2 \\ 25 & 0.2 < t < 0.5 \\ 0 & t > 0.5 \end{cases}$$

where a, b and c are real constants. (The current is given in Amps, the voltage in Volts and the time in seconds.) Determine the values of the constants:

a = ___10___A/s, b = ___–5.6___A and c = ___–0.6__A

At $t = 0.2$ s

$$i(0.2) = 5(0.2) - 4.6 = -3.6 \text{ A}$$

For $0.2 \le t \le 0.5$

$$i(t) = \frac{1}{2.5}\int_{0.2}^{t} 25\, d\tau - 3.6 = 10\,\tau \Big|_{0.2}^{t} - 3.6 = 10(t - 0.2) - 3.6 = 10t - 5.6 \text{ A}$$

At $t = 0.5$ s

$$i(0.5) = 10(0.5) - 5.6 = -0.6 \text{ A}$$

For $t \ge 0.5$

$$i(t) = \frac{1}{2.5}\int_{0.5}^{t} 0\, d\tau - 0.6 = -0.6$$

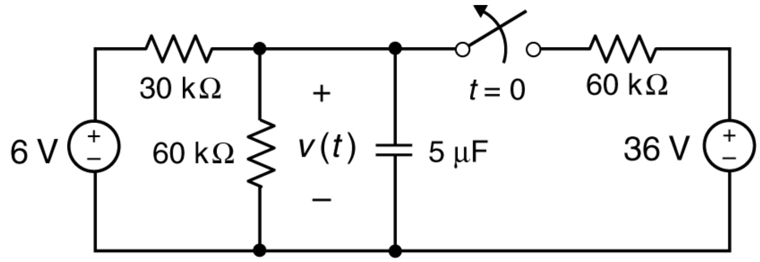
Checks:

At $t = 0.2$ s $\quad i(0.2) = 10(0.2) - 5.6 = -3.6 \text{ A} \quad \checkmark$

For $0.2 \le t \le 0.5$ $\quad v(t) = 2.5\frac{d}{dt}i(t) = 2.5\frac{d}{dt}(10t - 5.6) = 2.5(10) = 25 \text{ V} \quad \checkmark$

$$-0.6 - (-3.6) = i(0.5) - i(0.2) = \frac{1}{2.5}\int_{0.2}^{0.5} 25\, d\tau = 10(0.5 - 0.2) = 3 \text{ A} \quad \checkmark$$

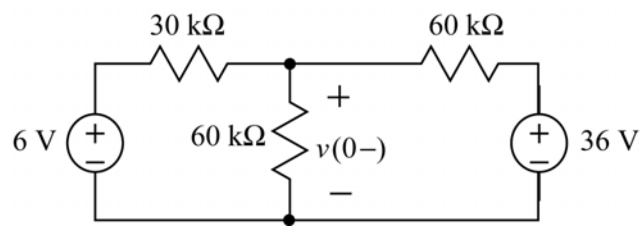
5. This circuit is at steady state when the switch opens at time $t = 0$.



The capacitor voltage is $v(t) = A + Be^{-at}$ for $t \ge 0$. Determine the values of the constants *A*, *B*, and *a*:

$A =$ _____4__ V, $B =$ __8___ V and $a =$ _____10___ s.

Solution:
Before $t = 0$, with the switch closed and the circuit at the steady state, the capacitor acts like an open circuit so we have

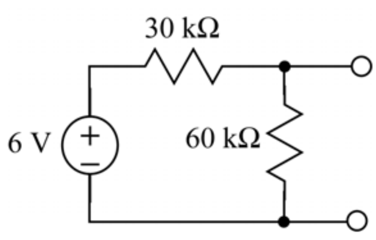


Using superposition

$$v(0-) = \frac{60 \,||\, 60}{30 + (60 \,||\, 60)} 6 + \frac{60 \,||\, 30}{60 + (60 \,||\, 30)} 36 = \left(\frac{1}{2}\right) 6 + \left(\frac{1}{4}\right) 36 = 12 \text{ V}$$

The capacitor voltage is continuous so $v(0+) = v(0-) = 12$ V.

After $t = 0$ the switch is open. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor:



$$v_{oc} = \frac{60}{60 + 30} 6 = 4 \text{ V}$$

$$R_t = 30 \,||\, 60 = 20 \text{ k}\Omega$$

The time constant is $\tau = R_t C = (20 \times 10^3)(5 \times 10^{-6}) = 0.1$ s so $\frac{1}{\tau} = 10 \ \frac{1}{\text{s}}$.

The capacitor voltage is given by

$$v(t) = (v(0+) - v_{oc}) e^{-t/\tau} + v_{oc} = (12 - 4) e^{-10t} + 4 = 4 + 8e^{-10t} \text{ V} \quad \text{for } t \ge 0$$

6. This circuit is at steady state before the switch closes at time $t = 0$. After the switch closes, the inductor current is given by

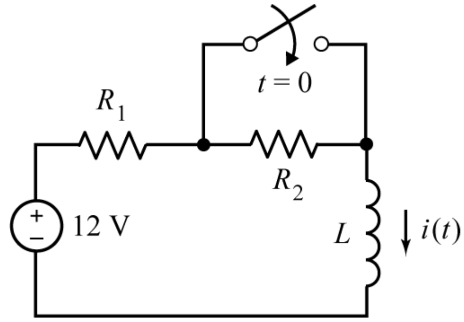
$$i(t) = 0.6 - 0.2e^{-5t} \text{ A} \quad \text{for } t \geq 0$$

Determine the values of R_1, R_2 and L:

$R_1 =$ ___20____ Ω , $R_2 =$ ___10____ Ω

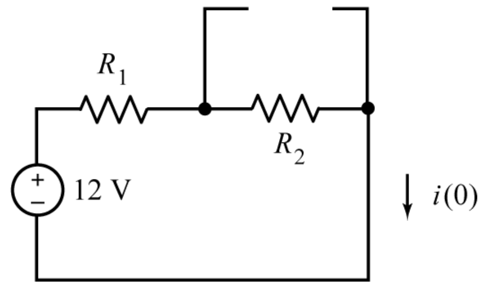
and

$L =$ ___4____ H



Solution:
The steady state current before the switch closes is equal to

$$i(0) = 0.6 - 0.2e^{-5(0)} = 0.4 \text{ A}.$$

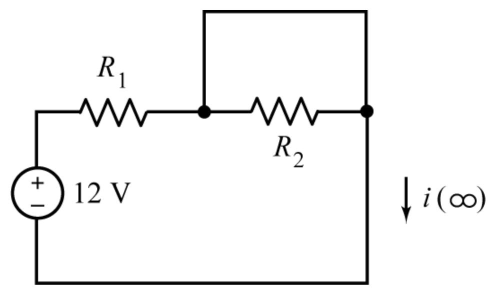


The inductor will act like a short circuit when this circuit is at steady state so

$$0.4 = i(0) = \frac{12}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 = 30 \ \Omega$$

After the switch has been open for a long time, the circuit will again be at steady state. The steady state inductor current will be

$$i(\infty) = 0.6 - 0.2e^{-5(\infty)} = 0.6 \text{ A}$$



The inductor will act like a short circuit when this circuit is at steady state so

$$0.6 = i(\infty) = \frac{12}{R_1} \quad \Rightarrow \quad R_1 = 20 \ \Omega$$

Then $R_2 = 10 \ \Omega$.

After the switch is closed, the Thevenin resistance of the part of the circuit connected to the inductor is $R_t = R_1$. Then

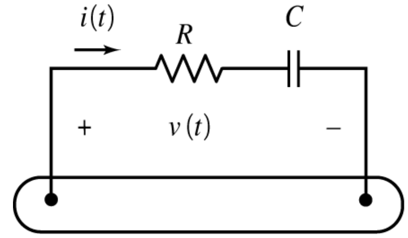
$$5 = \frac{1}{\tau} = \frac{R_t}{L} = \frac{R_1}{L} = \frac{20}{L} \quad \Rightarrow \quad L = 4 \text{ H}$$

7. The voltage and current for this circuit are given by

$$v(t) = 20 \cos(20t + 15°) \text{ V} \quad \text{and} \quad i(t) = 1.49 \cos(20t + 63°) \text{ A}$$

Determine the values of the resistance, R, and capacitance, C:

$R =$ ____9____ Ω and $C =$ ___5_____ mF.

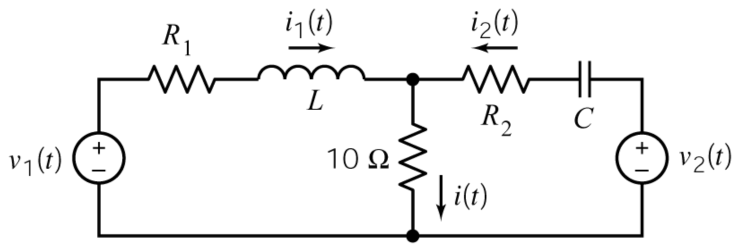


Solution:
In the frequency domain we have:

$$R - j\frac{1}{20C} = \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{20\angle 15°}{1.49\angle 63°} = \frac{20}{1.49}\angle\left(15° - 63°\right) = 13.42\angle -48° = 8.98 - j9.97\ \Omega$$

Equating real and imaginary parts gives $R = 9\ \Omega$ and $C = \dfrac{1}{20 \times 9.97} = 5\text{ mF}$.

8.



This circuit is at steady state. The voltage source voltages are given by

$$v_1(t) = 12\cos(2t - 90°)\text{ V} \quad \text{and} \quad v_2(t) = 5\cos(2t + 90°)\text{ V}$$

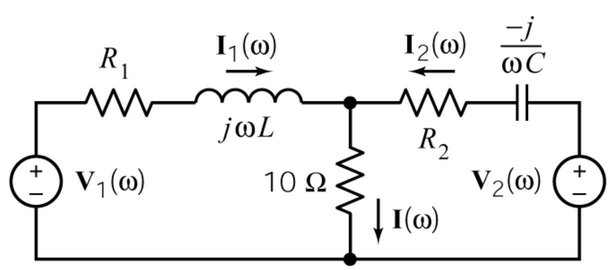
The currents are given by

$$i_1(t) = 744\cos(2t - 118°)\text{ mA},\ i_2(t) = 540.5\cos(2t + 100°)\text{ mA} \text{ and } i(t) = A\cos(2t - 164°)\text{ mA}$$

Determine the values of A, R_1, R_2, L and C:

$A =$ __460__ mA, $R_1 =$ __10__ Ω, $R_2 =$ __10__ Ω, $L =$ __6__ H and $C =$ __50__ mF.

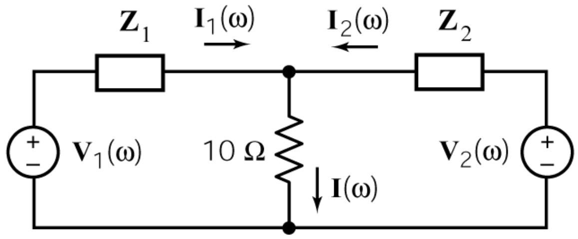
Solution:
Represent the circuit in the frequency domain using impedances and phasors:



$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.744\angle -118° + 0.5405\angle 100 &= \left(-0.349 - j0.657\right) + \left(-0.094 + j0.532\right) \\ &= \left(-0.349 - 0.094\right) + j\left(-0.657 + 0.532\right) \\ &= -0.443 - j\,0.125 \\ &= 0.460\angle -164°\end{aligned}$$

$$i(t) = 460\cos(2t - 164°)\ \text{mA}$$

Replacing series impedances by equivalent impedances gives



and

$$\mathbf{Z}_1 = R_1 + j\,\omega\,L$$

$$\mathbf{Z}_2 = R_2 - j\frac{1}{\omega\,C}$$

From KVL

$$\mathbf{Z}_1\mathbf{I}_1 + 10\,\mathbf{I} - \mathbf{V}_1 = 0 \quad \Rightarrow \quad \mathbf{Z}_1 = \frac{\mathbf{V}_1 - 10\,\mathbf{I}}{\mathbf{I}_1} = \frac{12\angle -90° - 10\left(0.460\angle -164°\right)}{0.744\angle -118°}$$

$$= \frac{-j12 - 10\left(-0.443 - j\,0.125\right)}{0.744\angle -118°}$$

$$= \frac{4.43 - j\,10.75}{0.744\angle -118°} = \frac{11.63\angle -67.6°}{0.744\angle -118°}$$

$$= 15.63\angle 50.4°$$

$$= 10 + j\,12\ \Omega$$

and

$$-\mathbf{Z}_2\mathbf{I}_2 + \mathbf{V}_2 - 10\,\mathbf{I} = 0 \quad \Rightarrow \quad \mathbf{Z}_2 = \frac{\mathbf{V}_2 - 10\,\mathbf{I}}{\mathbf{I}_2} = \frac{5\angle 90° - 10\left(0.460\angle -164°\right)}{0.5405\angle 100°}$$

$$= \frac{j\,5 - 10\left(-0.443 - j\,0.125\right)}{0.5405\angle 100°}$$

$$= \frac{4.43 + j\,6.25}{0.5405\angle 100°} = \frac{7.66\angle 54.7°}{0.5405\angle 100°}$$

$$= 14.14\angle -55.3°$$

$$= 10 - j\,10\ \Omega$$

Next

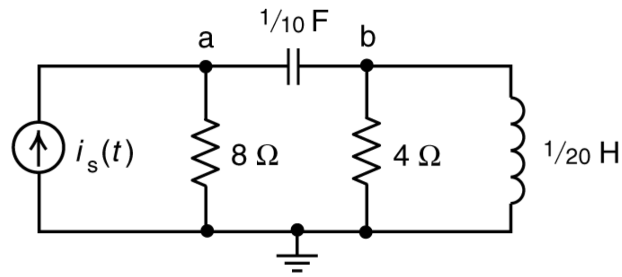
$$10 + j\,12 = R_1 + j\,\omega\,L = R_1 + j\,2\,L \quad \Rightarrow \quad R_1 = 10\ \Omega \quad \text{and} \quad L = \frac{12}{2} = 6\ \text{H}$$

and

$$10 - j\,10 = R_2 - j\frac{1}{\omega\,C} = R_2 - j\frac{1}{2\,C} \quad \Rightarrow \quad R_2 = 10\ \Omega \quad \text{and} \quad C = \frac{1}{2\left(10\right)} = 0.05\ \text{F}$$

9. The input this circuit is the current

$$i_s(t) = 2\cos(5t + 15°)\ \text{A}.$$



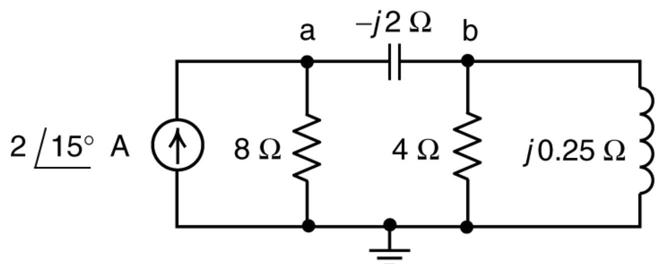
In the frequency domain, this circuit is represented by the node equation

$$\begin{bmatrix} d + j0.5 & -j0.5 \\ -j0.5 & 0.25 + je \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} 2\angle 15° \\ 0 \end{bmatrix}$$

where $\mathbf{V}_a$ and $\mathbf{V}_b$ are the phasor node voltages and d and e are real numbers. Determine the values of d and e.

$d =$ ___0.125___ Ω and $e =$ ____–3.5____ Ω

Solution: Represent the circuit in the frequency domain using impedances and phasors:



Apply KCL at node a to get

$$2\angle 15° = \frac{\mathbf{V}_a}{8} + \frac{\mathbf{V}_a - \mathbf{V}_b}{-j2} = (0.125 + j0.5)\mathbf{V}_a - (j0.5)\mathbf{V}_b$$

Apply KCL at node b to get

$$\frac{\mathbf{V}_a - \mathbf{V}_b}{-j2} = \frac{\mathbf{V}_b}{4} + \frac{\mathbf{V}_b}{j0.25} \quad \Rightarrow \quad j0.5(\mathbf{V}_a - \mathbf{V}_b) = (0.25 - j4)\mathbf{V}_b$$

$$\Rightarrow \quad 0 = -j0.5\mathbf{V}_a + (0.25 - j3.5)\mathbf{V}_b$$

Organize these equations into matrix form to get

$$\begin{bmatrix} 0.125 + j0.5 & -j0.5 \\ -j0.5 & 0.25 - j3.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} 2\angle 15° \\ 0 \end{bmatrix}$$

Compare this equation to the given node equation to see that d = 0.125 Ω and e = –3.5 Ω.