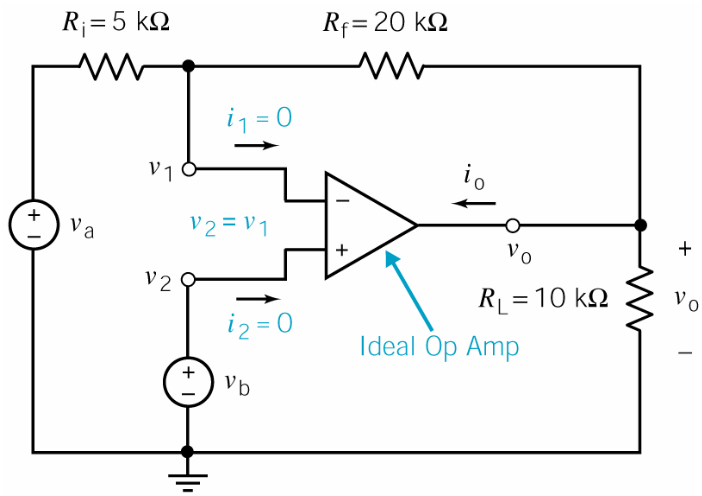

The inputs to this circuit is the voltage source voltages, v_a and v_b. The output is the voltage, v_o, across R_L.

One of the voltage sources causes $v_2 = v_b$. The op amp is ideal so $v_1 = v_2$. Consequently,

$$v_1 = v_2 = v_b$$

Apply KCL at the node connected to the inverting input of the op amp to get

$$\frac{v_a - v_1}{R_i} = i_1 + \frac{v_1 - v_o}{R_f} \quad \Rightarrow \quad \frac{v_a - v_b}{R_i} = \frac{v_b - v_o}{R_f} \quad \Rightarrow \quad R_f\left(v_a - v_b\right) = R_i\left(v_b - v_o\right)$$

$$\Rightarrow \quad v_o = \left(1 + \frac{R_f}{R_i}\right)v_b - \frac{R_f}{R_i}v_a = 5v_b - 4v_a$$

(Alternately, use superposition. When $v_b = 0$, the circuit is an inverting amplifier and $v_o = -\frac{R_f}{R_i}v_a$. When $v_a = 0$, the circuit is an noninverting amplifier and $v_o = \left(1 + \frac{R_f}{R_i}\right)v_b$. Then superposition gives $v_o = \left(1 + \frac{R_f}{R_i}\right)v_b - \frac{R_f}{R_i}v_a$.)

Next, consider the equation of a straight line.

$$y = m\,x + b$$

This equation is similar to

$$v_o = \left(1 + \frac{R_f}{R_i}\right) v_b - \frac{R_f}{R_i} v_a$$

We can match these equations in either of the following ways:

- Let $y = v_o$ and $x = v_b$. Then $m = 1 + \frac{R_f}{R_i}$ and $b = -\frac{R_f}{R_i} v_a = (1 - m) v_a$.
- Let $y = v_o$ and $x = v_a$. Then $m = -\frac{R_f}{R_i}$ and $b = \left(1 + \frac{R_f}{R_i}\right) v_b = (1 - m) v_b$.

Notice that in the first case the slope is greater than 1 when we choose $x = v_b$ and negative when we choose $x = v_a$.

Summary: Suppose we need to design a circuit so that the plot of the output versus the input is a given straight line. This circuit may, or may not, be able to do the job. It depends on the slope of the given line. Here's the design procedure:

1. Determine the slope, *m*, and intercept, *b*, of the straight line.
2. When *m* > 1,
    a. The input to circuit is v_b.
    b. Choose values of R_i and R_f so that $m = 1 + \frac{R_f}{R_i}$.
    c. Let $v_a = \frac{b}{(1 - m)}$.
3. When $m \le 0$
    a. The input to the circuit is v_a.
    b. Choose values of R_i and R_f so that $m = -\frac{R_f}{R_i}$.
    c. Let $v_b = \frac{b}{(1 - m)}$.
4. When $0 < m \le 1$ we'll need to find another circuit.

Example

Design a circuit having one input, v_s, and one output, v_o. The output is required to be related to the input by the equation

$$v_o = 10 v_s + 18$$

Solution:

The slope is m = 10 > 1 so we can use this circuit. Let $v_s = v_b$. Next

$$10 = 1 + \frac{R_f}{R_i} \quad \Rightarrow \quad \frac{R_f}{R_i} = 9$$

As a rule of thumb, we try to use resistances between a few kΩ and a few MΩ in op amp circuits. Let's pick $R_i = 20 \text{ k}\Omega$ and $R_f = 180 \text{ k}\Omega$. Finally,

$$v_a = \frac{b}{(1-m)} = \frac{18}{(1-10)} = -2 \text{ V}$$

Here's the circuit: