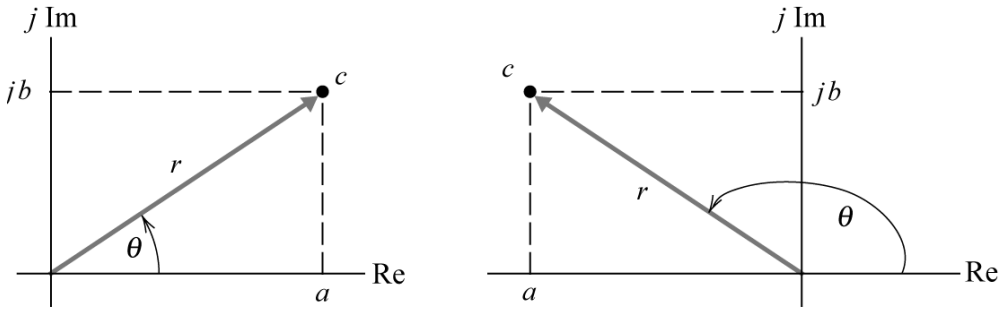
Complex Numbers

$$a + jb = re^{j\theta} = r\angle\theta$$

where $j = \sqrt{-1}$, $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2 + b^2}$

and $$\theta = \begin{cases} \tan^{-1}\left(\dfrac{b}{a}\right) & a > 0 \\ 180° - \tan^{-1}\left(\dfrac{b}{-a}\right) & a < 0 \end{cases}$$



$$c = a + jb = re^{j\theta}$$

$$1 = 1\angle 0°,\ \ j = 1\angle 90°,\ \ -1 = j^2 = 1\angle 180° = 1\angle -180°,\ \ -j = 1\angle -90°$$

Complex Arithmetic

$$(a + jb)(c + jd) = \left(Ae^{j\theta}\right)\left(Be^{j\phi}\right) = AB\angle(\theta + \phi)$$

$$\frac{a + jb}{c + jd} = \frac{Ae^{j\theta}}{Be^{j\phi}} = \frac{A}{B}\angle(\theta - \phi)$$

$$Ae^{j\theta} + Be^{j\phi} = (a + jb) + (c + jd) = (a + c) + j(b + d)$$

$$(a + jb)(c + jd) = (ac - bd) + j(ad + bc)$$

$$(a + jb)^* = a - jb,\ \ \left(Ae^{j\theta}\right)^* = Ae^{-j\theta},\ \ (A\angle\theta)^* = A\angle -\theta$$

$$(a + jb) + (a + jb)^* = 2a,\ (a + jb) - (a + jb)^* = j2b,\ \left(Ae^{j\theta}\right)\left(Ae^{j\theta}\right)^* = A^2$$