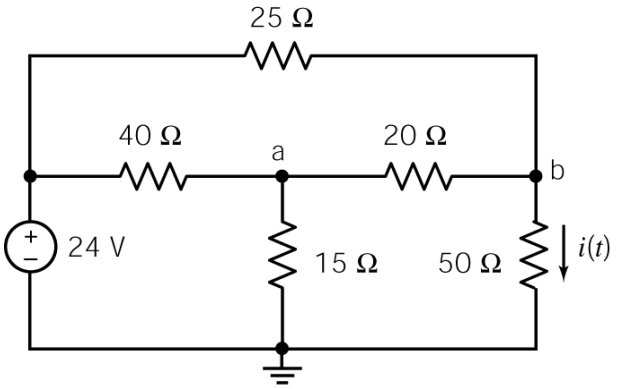
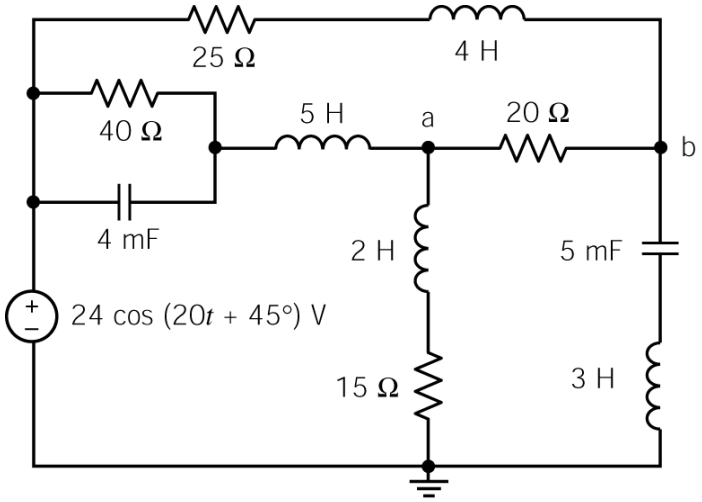
## Analyzing AC Circuits by Analogy with DC Circuits

Determine the node voltages of these two circuits.




(a)




(b)

## Solution

(a) The node equations are

$$\frac{24-v_a}{40}=\frac{v_a-v_b}{20}+\frac{v_a}{15}$$

$$\frac{24-v_b}{25}+\frac{v_a-v_b}{20}=\frac{v_b}{50}$$

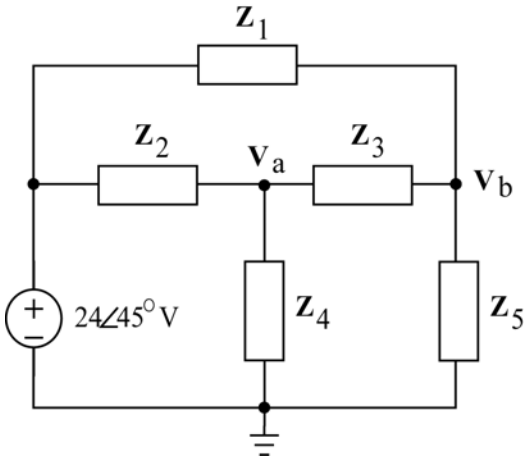
or

$$\begin{bmatrix}\frac{1}{40}+\frac{1}{20}+\frac{1}{15} & -\frac{1}{20}\\ -\frac{1}{20} & \frac{1}{25}+\frac{1}{20}+\frac{1}{50}\end{bmatrix}\begin{bmatrix}v_a\\ v_b\end{bmatrix}=\begin{bmatrix}\frac{24}{40}\\ \frac{24}{25}\end{bmatrix}$$

Solving using MATLAB gives

$$v_a = 8.713 \text{ V} \quad \text{and} \quad v_b = 12.69 \text{ V}$$

(b) Use phasors and impedances to represent the circuit in the frequency domain as




where

$$\mathbf{Z}_1 = 25 + j(20)4 = 25 + j80 = 83.82\angle 72.7^\circ\ \Omega$$

$$\mathbf{Z}_2 = \left(40 \,||\, \frac{1}{j(20)(0.004)}\right) + j(20)5 = 3.56 + j88.6 = 88.68\angle 87.7^\circ\ \Omega$$

$$\mathbf{Z}_3 = 20\ \Omega$$

$$\mathbf{Z}_4 = 15 + j(20)2 = 15 + j40 = 42.72\angle 69.4^\circ$$

$$\mathbf{Z}_5 = j(20)3 + \frac{1}{j(20)(0.005)} = j50 = 50\angle 90^\circ\ \Omega$$

The node equations are

$$\frac{24\angle 45^\circ - \mathbf{V}_a}{\mathbf{Z}_2} = \frac{\mathbf{V}_a}{\mathbf{Z}_4} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3}$$

$$\frac{24\angle 45^\circ - \mathbf{V}_b}{\mathbf{Z}_1} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3} = \frac{\mathbf{V}_b}{\mathbf{Z}_5}$$

$$\begin{bmatrix} \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4} & -\frac{1}{\mathbf{Z}_3} \\ -\frac{1}{\mathbf{Z}_3} & \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_5} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{24\angle 45^\circ}{\mathbf{Z}_2} \\ \frac{24\angle 45^\circ}{\mathbf{Z}_1} \end{bmatrix}$$

Solving using MATLAB gives

$$\mathbf{V}_a = 7.89\angle 44.0^\circ$$

$$\mathbf{V}_b = 8.45\angle 45.1^\circ$$

so

$$v_a(t) = 7.89\cos(20t + 44^\circ)\ \text{V}$$

$$v_a(t) = 8.45\cos(20t + 45.1)\ \text{V}$$

## MATLAB

### The dc Circuit

```
>> A = [1/40 + 1/20 + 1/15     -1/20;
             -1/20         1/25 + 1/20 + 1/50];
>> b = [24/40; 24/25];
>> v=A\b

v =

    8.7134
   12.6879
```

### The ac Circuit

```
>> Z1 = 25 + j*20*4;
>> Z2 = 88.7*exp(j*87.7*pi/180);
>> Z3 = 20;
>> Z4 = 15 + j*20*2;
>> Z5 = j*20*3 + 1/j/20/0.005;
>> A = [1/Z2 + 1/Z3 + 1/Z4       -1/Z3;
             -1/Z3           1/Z1 + 1/Z3 + 1/Z5];
>> b = [24*exp(j*45*pi/180)/Z2; 24*exp(j*45*pi/180)/Z1];
>> v = A\b;
>> abs(v)

ans =

    7.8877
    8.9481

>> angle(v)*180/pi

ans =

   44.0246
   45.1001
```