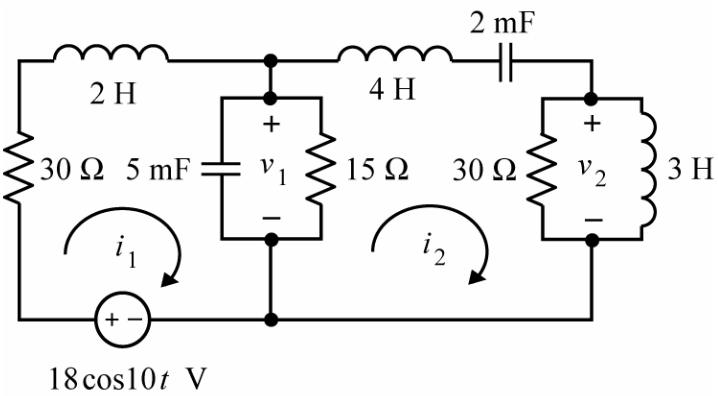
Analysis of AC Circuits

Example 1: Determine the node voltages, $v_1(t)$ and $v_2(t)$, and the mesh currents, $i_1(t)$ and $i_2(t)$, for this circuit.

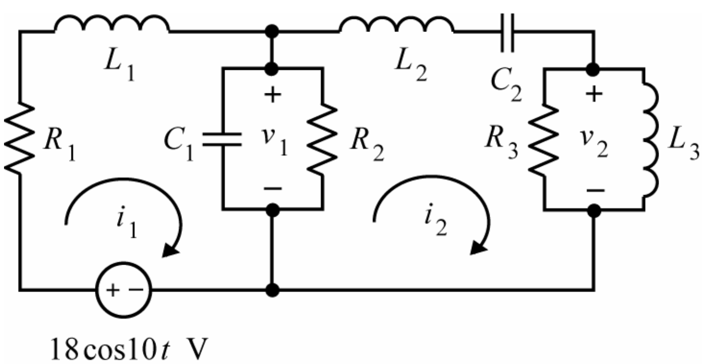


Example 2: In this circuit, the node voltages are

$$v_1(t) = 3.318\cos(10t - 39.3°)\ \text{V} \text{ and } v_2(t) = 4.452\cos(10t - 12.7°)\ \text{V},$$

and the mesh currents are

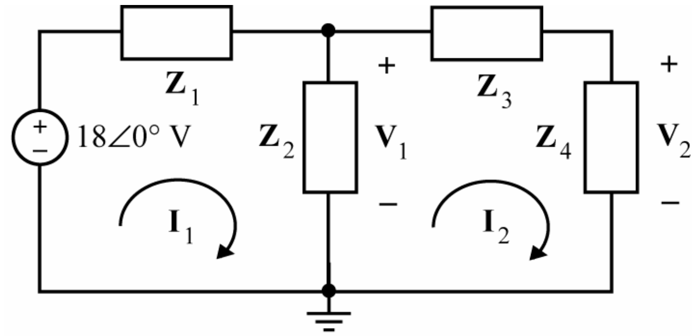
$$i_1(t) = 0.4319\cos(10t - 25.9°)\ \text{A} \text{ and } i_2(t) = 0.2099\cos(10t - 57.7°)\ \text{A}.$$



Determine the values of the resistances, inductances and capacitances.

Solutions

Example 1: Represent the circuit in the frequency domain using impedances and phasors:



where

$$\mathbf{Z}_1 = 30 + j(10)(2) = 30 + j20\ \Omega, \qquad \mathbf{Z}_2 = \frac{15\dfrac{1}{j(10)(0.005)}}{15 + \dfrac{1}{j(10)(0.005)}} = \frac{15(-j20)}{15 - j20} = 9.6 - j7.2\ \Omega$$

$$\mathbf{Z}_3 = j(10)(4) + \frac{1}{j(10)(0.002)} = -j10\ \Omega \quad \text{and} \quad \mathbf{Z}_4 = \frac{(30)\,j(10)(3)}{30 + j(10)(3)} = 15 + j15\ \Omega$$

The node equations are

$$\frac{18\angle 0 - \mathbf{V}_1}{\mathbf{Z}_1} = \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} \quad \Rightarrow \quad (\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3)\mathbf{V}_1 - \mathbf{Z}_1\mathbf{Z}_2\mathbf{V}_2 = \mathbf{Z}_2\mathbf{Z}_3(18\angle 0)$$

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} = \frac{\mathbf{V}_2}{\mathbf{Z}_4} \quad \Rightarrow \quad -\mathbf{Z}_4\mathbf{V}_1 + (\mathbf{Z}_3 + \mathbf{Z}_4)\mathbf{V}_2 = 0$$

Substituting the values of the impedances and writing the equations in matrix form gives

$$\begin{bmatrix} 560 - j420 & -432 + j24 \\ -15 - j15 & 15 + j5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} -1296 - j1728 \\ 0 \end{bmatrix}$$

Solving, for example using MATLAB, we get

$$\mathbf{V}_1 = 3.318\angle -39.3^\circ\ \text{V} \quad \text{and} \quad \mathbf{V}_2 = 4.452\angle -12.7^\circ\ \text{V}$$

Back in the time domain, the node voltages are

$$v_1(t) = 3.318\cos(10t - 39.3^\circ)\ \text{V} \ \text{and} \ v_2(t) = 4.452\cos(10t - 12.7^\circ)\ \text{V}$$

The mesh equations are

$$\mathbf{Z}_1\mathbf{I}_1+\mathbf{Z}_2\left(\mathbf{I}_1-\mathbf{I}_2\right)-18\angle 0°=0 \quad \Rightarrow \quad \left(\mathbf{Z}_1+\mathbf{Z}_2\right)\mathbf{I}_1-\mathbf{Z}_2\mathbf{I}_2=18\angle 0°$$

$$\mathbf{Z}_3\mathbf{I}_2+\mathbf{Z}_4\mathbf{I}_2-\mathbf{Z}_2\left(\mathbf{I}_1-\mathbf{I}_2\right)=0 \quad \Rightarrow \quad -\mathbf{Z}_2\mathbf{I}_1+\left(\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4\right)\mathbf{I}_2=0$$

Substituting the values of the impedances and writing the equations in matrix form gives

$$\begin{bmatrix} 39.6-j12.8 & -9.6+j7.2 \\ -9.6+j7.2 & 24.6-j2.2 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}=\begin{bmatrix} 18 \\ 0 \end{bmatrix}$$

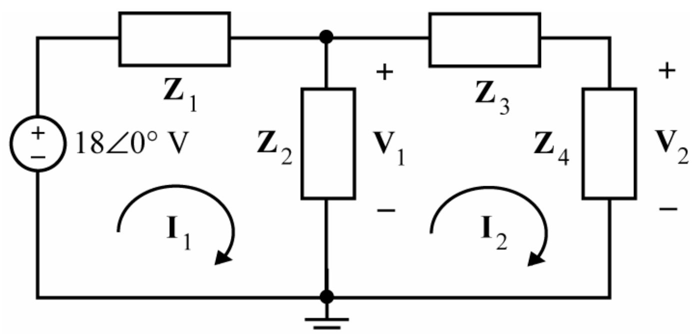
Solving, for example using MATLAB, we get

$$\mathbf{I}_1=0.4319\angle-25.9° \text{ A} \quad \text{and} \quad \mathbf{I}_2=0.2099\angle-57.7° \text{ A}$$

Back in the time domain, the mesh currents are

$$i_1(t)=0.4319\cos\left(10t-25.9°\right) \text{ A} \;\text{ and }\; i_2(t)=0.2099\cos\left(10t-57.7°\right) \text{ A}$$

Example 2: Represent the circuit in the frequency domain using impedances and phasors:



where

$$\mathbf{Z}_1=R_1+j\omega L_1, \qquad \frac{1}{\mathbf{Z}_2}=\mathbf{Y}_2=\frac{1}{R_2}+j\omega C_1$$

$$\mathbf{Z}_3=j\omega L_2+\frac{1}{j\omega C_2}=j\left(\omega L_2-\frac{1}{\omega C_2}\right) \quad \text{and} \quad \frac{1}{\mathbf{Z}_4}=\mathbf{Y}_4=\frac{1}{R_3}+\frac{1}{j\omega L_3}=\frac{1}{R_3}-j\frac{1}{\omega L_3}$$

and where

$$\mathbf{V}_1=3.318\angle-39.3° \text{ V}, \quad \mathbf{V}_2=4.452\angle-12.7° \text{ V}$$

$$\mathbf{I}_1=0.4319\angle-25.9° \text{ A} \quad \text{and} \quad \mathbf{I}_2=0.2099\angle-57.7° \text{ A}$$

Recalling that ω = 10 rad/s, we have

$$R_1 + j10L_1 = \mathbf{Z}_1 = \frac{18\angle 0° - \mathbf{V}_1}{\mathbf{I}_1} = \frac{18\angle 0° - 3.318\angle -39.3°}{0.4319\angle -25.9°} = 30 + j20\ \Omega$$

$$\frac{1}{R_2} + j10C_1 = \mathbf{Y}_2 = \frac{\mathbf{V}_2}{\mathbf{I}_1 - \mathbf{I}_2} = \frac{4.452\angle -12.7°}{0.4319\angle -25.9° - 0.2099\angle -57.7°} = 0.0667 + j0.05\ \text{S}$$

$$j\left(10L_2 - \frac{1}{10C_2}\right) = \mathbf{Z}_3 = \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{I}_2} = \frac{3.318\angle -39.3° - 4.452\angle -12.7°}{0.2099\angle -57.7°} = -j10\ \Omega$$

Consequently:

$$\frac{1}{R_3} - j\frac{1}{10L_3} = \mathbf{Y}_4 = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{0.2099\angle -57.7°}{4.452\angle -12.7°} = 0.0333 - j0.0333\ \text{S}$$

$$R_1 = 30\ \Omega,\ L_1 = \frac{20}{10} = 2\ \text{H},\ R_2 = \frac{1}{0.0667} = 15\ \Omega,\ C_1 = \frac{0.05}{10} = 5\ \text{mF},$$

$$10L_2 - \frac{1}{10C_2} = -10 \quad \Rightarrow \quad L_2 + 1 = \frac{1}{100C_2}, \text{ e.g. } C_2 = 2\ \text{mF and } L_2 = \frac{1}{100(0.002)} - 1 = 4\ \text{H},$$

$$R_3 = \frac{1}{0.0333} = 30\ \Omega \quad \text{and} \quad L_3 = \frac{1}{(10)0.0333} = 3\ \text{H}$$