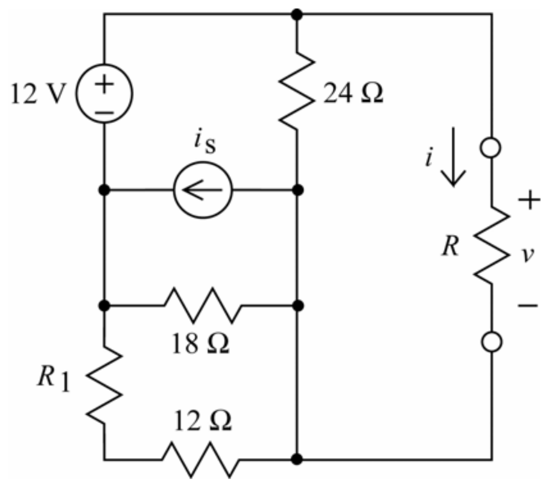
Determining a Thevenin Equivalent Circuit from Measured Data

Example:
Here's a circuit and some corresponding data. Two resistances, R_1 and R, and the current source current are unspecified. The tabulated data provides values of the current, i, and voltage, v, corresponding to several values of the resistance R.
(a) Consider replacing the part of the circuit connected to the resistor R by a Thevenin equivalent circuit. Use the data in rows 2 and 3 of the table to find the values of R_t and v_{oc}, the Thevenin resistance and the open circuit voltage.
(b) Use the results of part (a) to verify that the tabulated data is consistent.
(c) Fill in the blanks in the table.
(d) Determine the values of R_1 and i_s.



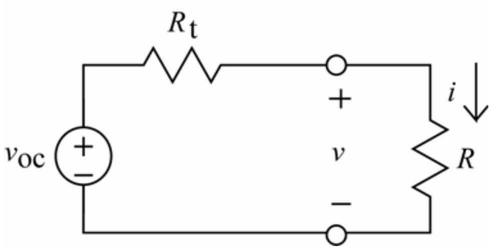
(a)

R, Ω	i, A	v, V
0	3	0
10	1.333	13.33
20	0.857	17.14
40	0.5	?
80	?	21.82

(b)

Solution

(a)



KVL gives

$$v_{oc} = (R_t + R)i$$

from row 2

$$v_{oc} = (R_t + 10)(1.333)$$

from row 3

$$v_{oc} = (R_t + 20)(0.857)$$

So

$$(R_t + 10)(1.333) = (R_t + 20)(0.857)$$

$$28(R_t + 10) = 18(R_t + 20)$$

Solving gives

$$10R_t = 360 - 280 = 80 \quad \Rightarrow \quad R_t = 8\ \Omega$$

and

$$v_{oc} = (8 + 10)(1.333) = 24\ \text{V}$$

(b)

$$i = \frac{v_{oc}}{R_t + R} = \frac{24}{8 + R} \quad \text{and} \quad v = \frac{R}{R + R_t} v_{oc} = \frac{24R}{R + 8}$$

When R = 0, i = 3 A, and v = 0 V.

When R = 40 Ω, $i = \frac{1}{2}$ A.

When R = 80 Ω, $v = \frac{24(80)}{88} = \frac{240}{11} = 21.82$.

These are the values given in the tabulated data so the data is consistent.

(c) When R = 40 Ω, $v = \frac{24(40)}{48} = 20$ V.

When R = 80 Ω, $i = \frac{24}{88} = 0.2727$ A.

(d) First

$$8 = R_t = 24 \,||\, 18 \,||\, (R_1 + 12) \quad \Rightarrow \quad R_1 = 24\ \Omega$$

the, using superposition,

$$24 = v_{oc} = \frac{24}{24 + (18 \,||\, (R_1 + 12))} 12 + (24 \,||\, 18(R_1 + 12)) i_s = 8 + 8 i_s \quad \Rightarrow \quad i_s = 2\ \text{A}$$