An Experiment

A Circuit
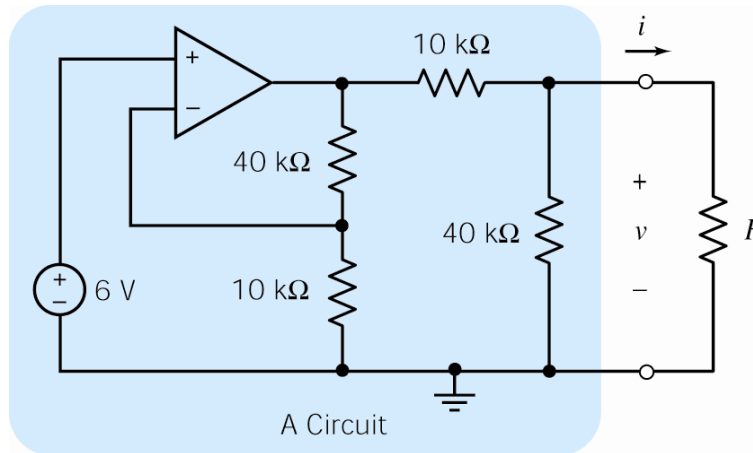
Consider this experiment. We connect a resistor to the terminals of a circuit, then measure the resistor voltage and current. Next, we change the resistor and measure the voltage and current again. After some trial and error, we collect the following date:

R, kΩ	0	1	2	5	10	20	50	∞
i, mA	3	2.667	2.4	1.846	1.33	0.857	0.414	0
v, V	0	2.667	4.8	9.231	13.33	17.143	20.69	24

In the case $R = \infty$, we have connected an open circuit across the terminals. As expected, $i = 0$. In this case the voltage is referred to as the "open circuit voltage", denoted as v_{oc}. We have $v_{oc} = 24$ V. In the case $R = 0$, we have connected a short circuit across the terminals. As expected, $v = 0$. In this case the current is referred to as the "short circuit current", denoted as i_{sc}. We have $i_{sc} = 3$ A.

Perhaps it would be helpful to plot the data:
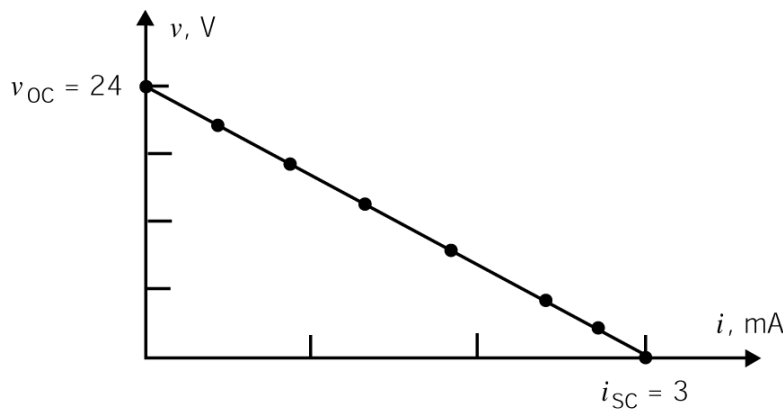
The data lies on a straight line that connects the points $(i_{sc}, 0)$ and $(0, v_{oc})$!

The slope of the straight line is $-\dfrac{v_{oc}}{i_{sc}}$. $\dfrac{v_{oc}}{i_{sc}}$ has units of Ω. It's convenient to define R_t as
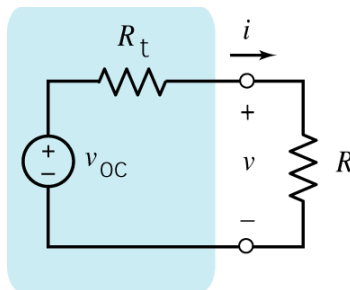
$$R_t = \frac{v_{oc}}{i_{sc}}$$

The equation of the straight line is

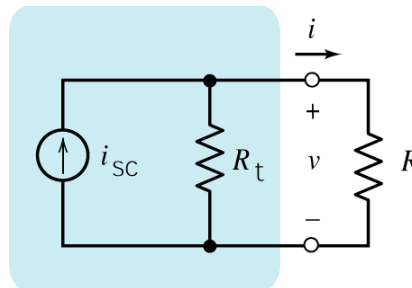$$v = \left(-\frac{v_{oc}}{i_{sc}}\right)i + v_{oc}$$

or

$$v = -R_t\,i + v_{oc}$$

Next, consider this circuit:

Apply KVL to get

$$R_t\,i + v - v_{oc} = 0 \quad \Rightarrow \quad v = -R_t\,i + v_{oc} = \left(-\frac{v_{oc}}{i_{sc}}\right)i + v_{oc}$$
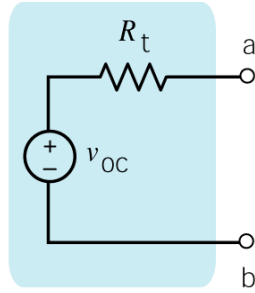
Next, consider this circuit:

Apply KCL to get

$$i_{sc} = \frac{v}{R_t} + i = 0 \quad \Rightarrow \quad v = -R_t\,i + R_t\,i_{sc} = \left(-\frac{v_{oc}}{i_{sc}}\right)i + v_{oc}$$
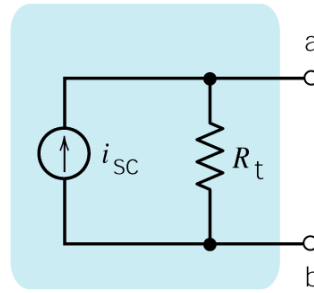
The three circuits shaded in blue are each represents by the equation that describes our data. Any one of them could have generated our data!

We cannot distinguish between these circuits use our data. In this sense, these circuits are equivalent.
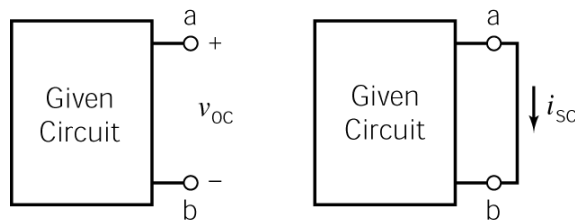
The equivalent circuits have names:
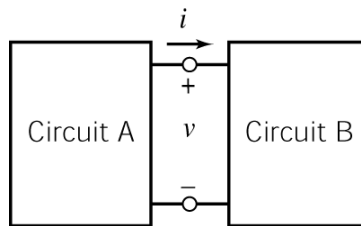

Thevenin Equivalent Circuit   Norton Equivalent Circuit

These equivalent circuits are characterized by three parameters: v_{oc}, i_{sc} and R_t.

The parameters v_{oc} and i_{sc} are calculated or measured from a given circuit as

Suppose we take any circuit and separate it into two parts that are connected at a single pair of terminals:
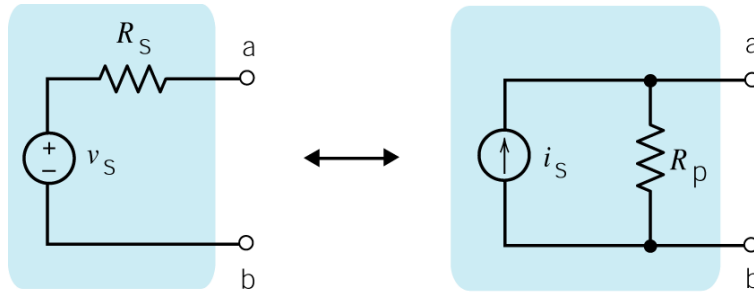

No current or voltage in Circuit B will change if Circuit A is replaced by its Thevenin equivalent circuit or by its Norton equivalent circuit.

Suppose we knew the value of every current and voltage in Circuit B. We would not be able to determine if Circuit B was connected to

- Circuit A
- The Thevenin Equivalent Circuit of Circuit A

or

- The Norton Equivalent Circuit of Circuit A

The Thevenin and Norton equivalent circuits are equivalent to each other. Consequently, we can replace a series voltage source and resistor with a parallel current source and resistor, without changing the voltage or current of any other element of the circuit. (Similarly, we can replace a parallel current source and resistor with a series voltage source and resistor, without changing the voltage or current of any other element of the circuit.) Such a substitution is called a source transformation.

$$R_s = R_p$$
$$v_s = R_p i_s$$

$$R_p = R_s$$
$$i_s = \frac{v_s}{R_s}$$