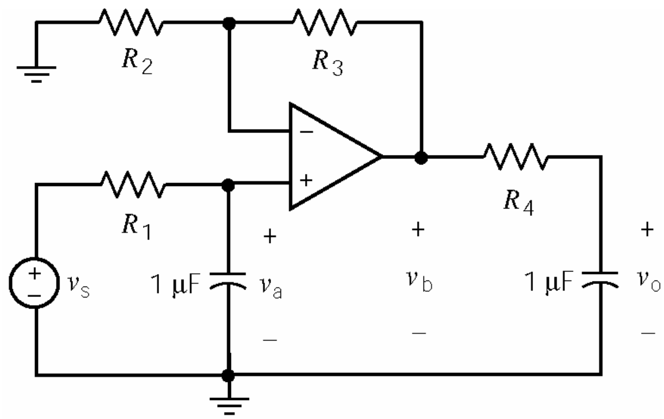
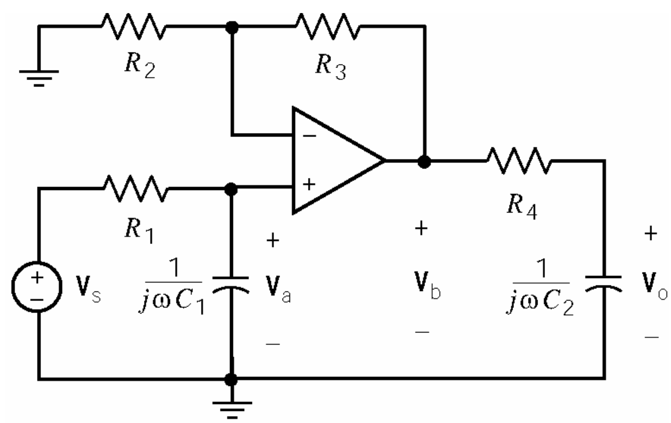
Example:
The input to this circuit is the voltage of the voltage source, v_s. The output of the circuit is the capacitor voltage, v_o.



Determine the values of the resistances R_1, R_2, R_3 and R_4 required to cause the network function of the circuit to be

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{21}{\left(1+j\frac{\omega}{5}\right)\left(1+j\frac{\omega}{200}\right)}$$

Solution:
Represent the circuit in the frequency domain.



Apply KCL at the top node of the left capacitor, C_1, to get

$$\frac{\mathbf{V}_a-\mathbf{V}_s}{R_1}+j\omega C_1\mathbf{V}_a=0 \quad\Rightarrow\quad \mathbf{V}_a=\frac{1}{1+j\omega C_1 R_1}\mathbf{V}_s$$

The op amp, together with resistors R_2 and R_3, comprise a noninverting amplifier so

$$\mathbf{V}_{\mathrm{b}} = \left(1 + \frac{R_3}{R_2}\right)\mathbf{V}_{\mathrm{a}}$$

(Alternately, this equation can be obtained by applying KCL at the inverting input node of the op amp.) Apply KCL at the top node of the right capacitor, C_2, to get

$$\frac{\mathbf{V}_{\mathrm{o}} - \mathbf{V}_{\mathrm{b}}}{R_4} + j\omega C_2 \mathbf{V}_{\mathrm{o}} = 0 \quad \Rightarrow \quad \mathbf{V}_{\mathrm{o}} = \frac{1}{1 + j\omega C_2 R_4}\mathbf{V}_{\mathrm{b}}$$

Combining these equations gives

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_{\mathrm{o}}(\omega)}{\mathbf{V}_{\mathrm{s}}(\omega)} = \frac{1 + \dfrac{R_3}{R_2}}{\left(1 + j\omega C_1 R_1\right)\left(1 + j\omega C_2 R_4\right)}$$

Comparing to the specified network function gives

$$\frac{1 + \dfrac{R_3}{R_2}}{\left(1 + j\omega C_1 R_1\right)\left(1 + j\omega C_2 R_4\right)} = \frac{21}{\left(1 + j\dfrac{\omega}{5}\right)\left(1 + j\dfrac{\omega}{200}\right)}$$

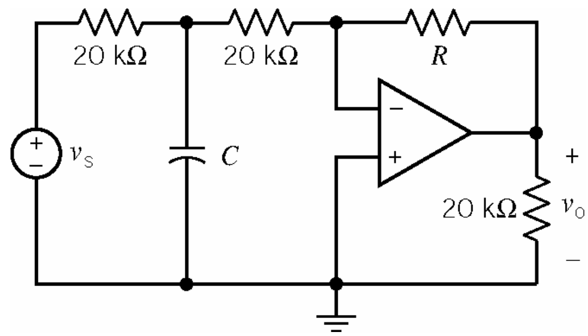
The solution is not unique. For example, we can require

$$1 + \frac{R_3}{R_2} = 21, \; C_1 R_1 = \frac{1}{5} = 0.2, \; C_2 R_4 = \frac{1}{200} = 0.005$$

With the given values of capacitance, and choosing R_2 = 10 kΩ, we have

R_1 = 200 kΩ, R_2 = 10 kΩ, R_3 = 200 kΩ and R_4 = 50 kΩ

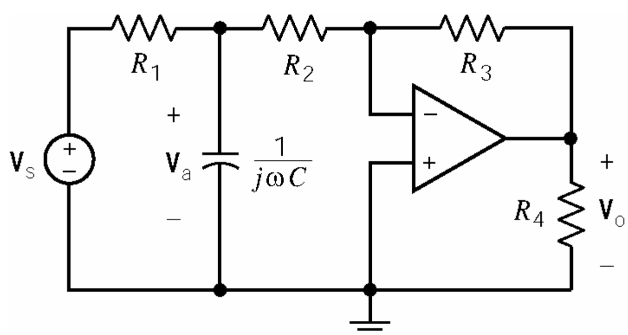
Example:
The input to this circuit is the voltage of the voltage source, v_s. The output of the circuit is the resistor voltage, v_o.



Specify values for R and C that cause the network function of the circuit to be

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{-8}{1 + j\frac{\omega}{250}}$$

Solution:
Represent the circuit in the frequency domain.



The node equations are

$$\frac{\mathbf{V}_a - \mathbf{V}_s}{R_1} + \frac{\mathbf{V}_a}{\frac{1}{j\omega C}} + \frac{\mathbf{V}_a}{R_2} = 0 \quad \Rightarrow \quad \mathbf{V}_a = \frac{R_2}{R_1 + R_2 + j\omega C R_1 R_2}\mathbf{V}_s$$

and

$$\frac{\mathbf{V}_a}{R_2} + \frac{\mathbf{V}_o}{R_3} = 0 \quad \Rightarrow \quad \mathbf{V}_o = -\frac{R_3}{R_2}\mathbf{V}_a$$

The network function is

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\frac{R_3}{R_2}R_2}{R_1 + R_2 + j\omega C R_1 R_2} = \frac{-\frac{R_3}{R_1 + R_2}}{1 + j\omega C\frac{R_1 R_2}{R_1 + R_2}}$$

Using the given values for R_1 and R_2 and letting $R_3 = R$ gives

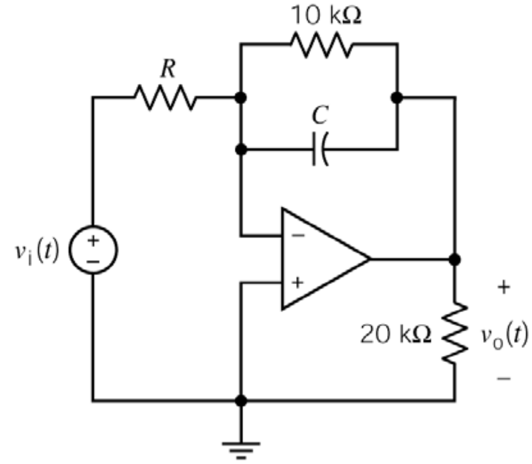
$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\dfrac{R}{4\times10^4}}{1 + j\omega C\left(10^4\right)}$$

Comparing this network function to the specified network function gives

$$C\left(10^4\right) = \frac{1}{250} \quad \Rightarrow \quad C = 0.4\ \mu\text{F} \ \text{ and } \ \frac{R}{4\times10^4} = 8 \quad \Rightarrow \quad R = 320\ \text{k}\Omega$$

Example

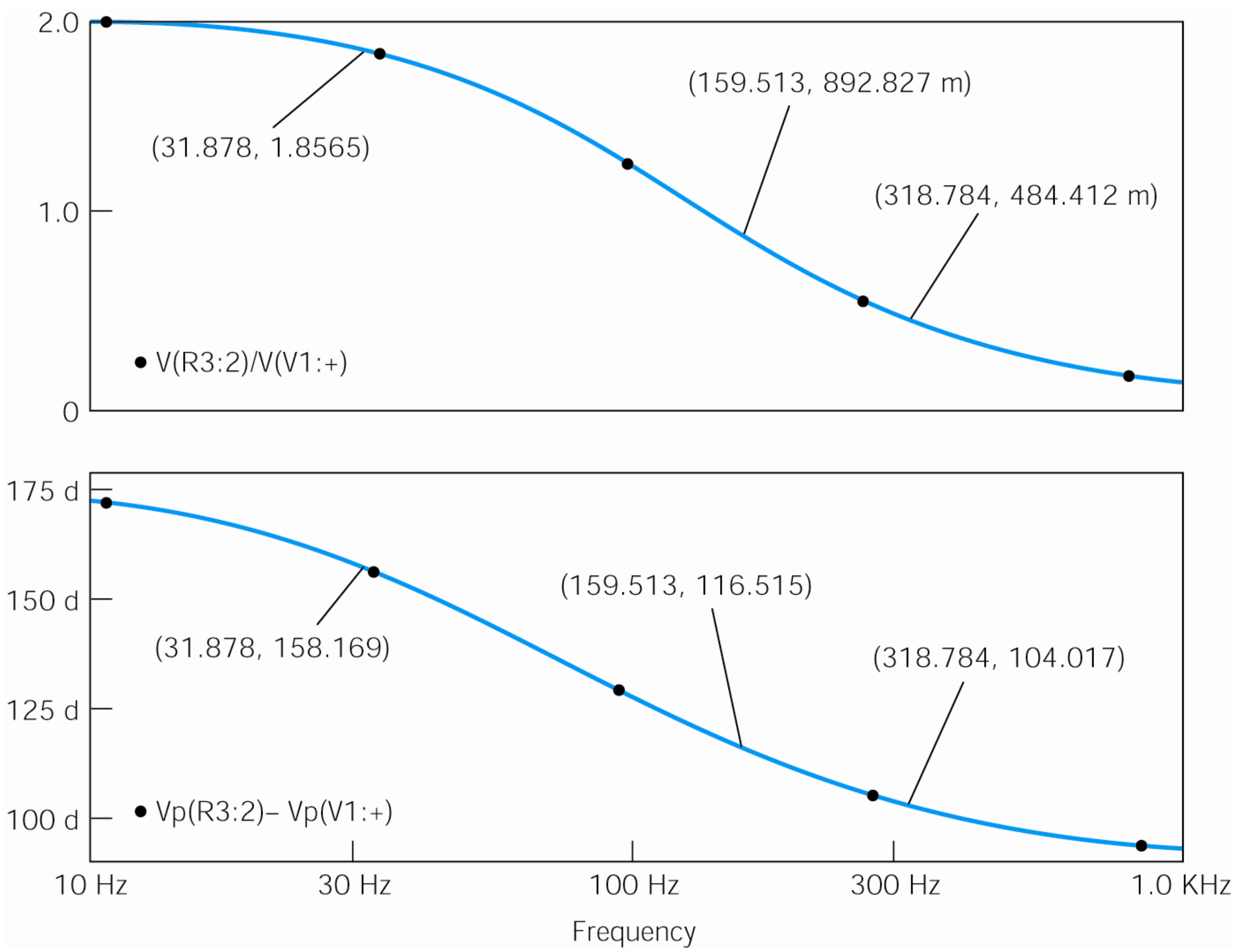
Consider this circuit and its frequency response, shown below.



The frequency response plots were made using PSpice and Probe. V(R3:2) and Vp(R3:2) denote the magnitude and angle of the phasor corresponding to $v_o(t)$. V(V1:+) and Vp(V1:+) denote the magnitude and angle of the phasor corresponding to $v_i(t)$. Hence V(R3:2)/ V(V1:+) is the gain of the circuit and Vp(R3:2)- Vp(V1:+) is the phase shift of the circuit.

Determine values for *R* and *C* required to make the circuit correspond to the frequency response.

Hint: PSpice and Probe use m for milli or 10^{-3}. Hence, the label (159.513, 892.827m) indicates that the gain of the circuit is $892.827 * 10^{-3} = 0.892827$ at a frequency of 159.513 Hertz ≈ 1000 rad/sec.



Solution:

$$\mathbf{H}(\omega) = -\frac{\frac{10^4}{R}}{1 + j\omega C\,10^4} = \frac{\frac{10^4}{R}}{\sqrt{1+\left(\omega C\,10^4\right)^2}} \angle -\tan^{-1}\left(\omega C\,10^4\right)$$

When ω = 200 rad/sec = 31.83 Hertz

$$1.8565\angle 158° = \frac{\frac{10^4}{R}}{\sqrt{1+\left(\omega C\,10^4\right)^2}} \angle -\tan^{-1}\left(\omega C\,10^4\right)$$

Equating phase shifts gives

$$\omega C\,10^4 = 10^3 \frac{C\,R\,10^4}{R+10^4} = \tan(22°) = 0.404 \quad \Rightarrow \quad C = 0.2\ \mu\text{F}$$

Equating gains gives

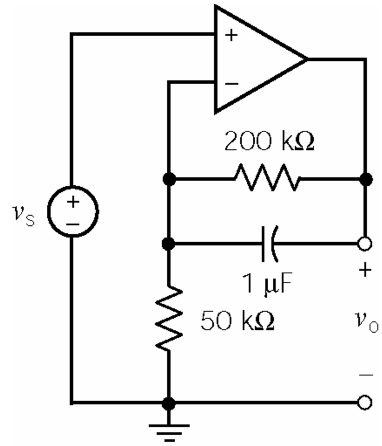
$$1.8565 = \frac{\frac{10^4}{R}}{\sqrt{1+\left(\omega C\,10^4\right)^2}} = \frac{\frac{10^4}{R}}{\sqrt{1+\left(0.404\right)^2}} \quad \Rightarrow \quad R = 5\ \text{k}\Omega$$

Example:

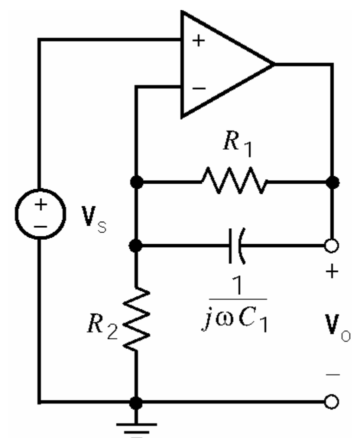
The input to this circuit is the voltage of the voltage source, v_s. The output of the circuit is the voltage, v_o. Determine the network function,

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

of the circuit.



Solution:
Represent the circuit in the frequency domain. Apply KCL at the inverting input node of the op amp to get



$$\frac{\mathbf{V}_o-\mathbf{V}_s}{R_1}+j\omega C_1\left(\mathbf{V}_o-\mathbf{V}_s\right)+\frac{\mathbf{V}_s}{R_2}=0$$

or

$$\left(R_1+R_2+j\omega C_1 R_1 R_2\right)\mathbf{V}_s=\left(R_2+j\omega C_1 R_1 R_2\right)\mathbf{V}_o$$

so

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{R_1+R_2+j\omega C_1 R_1 R_2}{R_2+j\omega C_1 R_1 R_2}=\frac{R_1+R_2}{R_2}\times\frac{1+j\omega C_1\dfrac{R_1 R_2}{R_1+R_2}}{1+j\omega C_1 R_1}$$

With the given values

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{25+j\omega}{5+j\omega}=5\,\frac{1+j\dfrac{\omega}{25}}{1+j\dfrac{\omega}{5}}$$