Example: Four similar circuits are shown below.. In each of these circuits
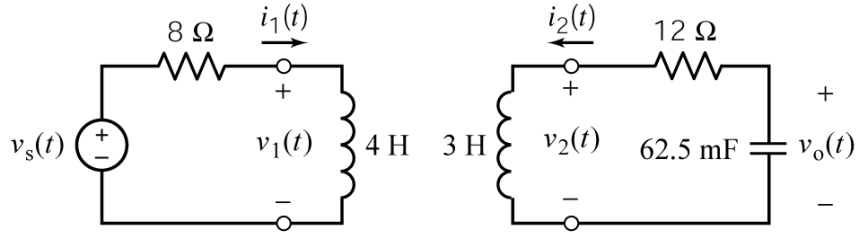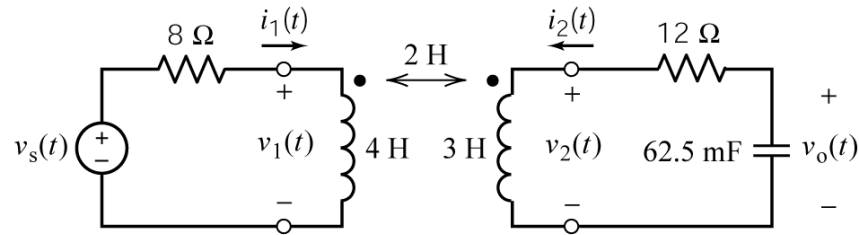
$$v_s(t) = 5 \cos (4t + 45°) \text{ V}.$$

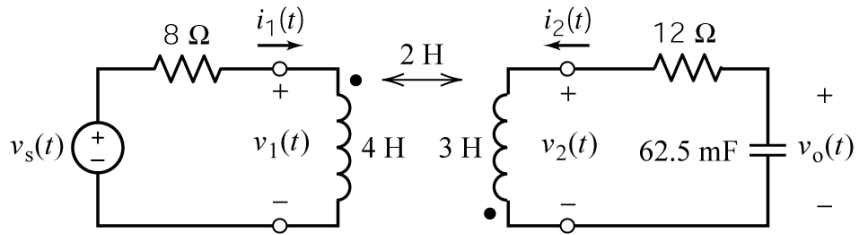Determine $v_2 (t)$ for each of the circuits.

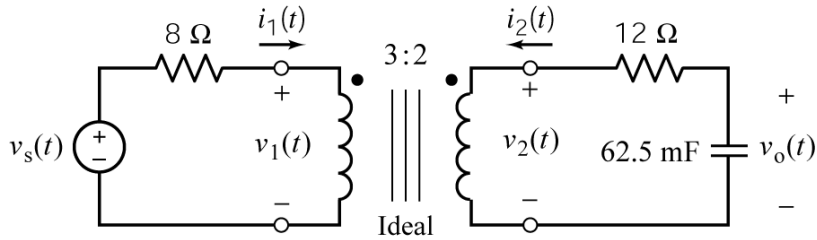

(a)
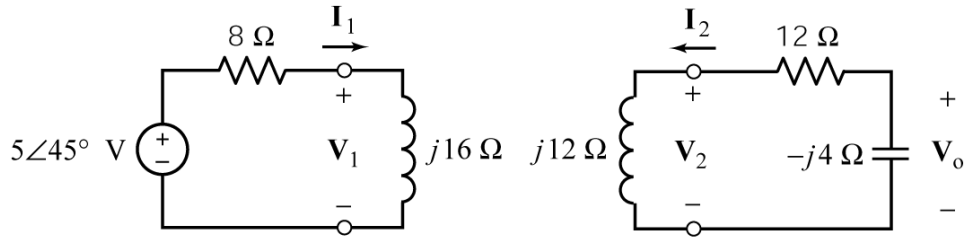

(b)

(c)

(d)

Solution:

(a)

Coil voltages:

$$\mathbf{V_1} = j16\,\mathbf{I_1}$$
$$\mathbf{V_2} = j12\,\mathbf{I_2}$$

Mesh equations:

$$8\,\mathbf{I_1} + \mathbf{V_1} - 5\angle 45° = 0$$
$$-(12 - j4)\,\mathbf{I_2} - j\mathbf{V_2} = 0$$

Substitute the coil voltages into the mesh equations and do some algebra:

$$8\,\mathbf{I_1} + j16\,\mathbf{I_1} = 5\angle 45° \quad \Rightarrow \quad \mathbf{I_1} = 0.28\angle -18.4°$$
$$-(12 - j4)\,\mathbf{I_2} - j12\,\mathbf{I_2} = 0 \quad \Rightarrow \quad (-12 + j8)\,\mathbf{I_2} = 0 \quad \Rightarrow \quad \mathbf{I_2} = 0$$

$$\mathbf{V_o} = -\left(-j4\,\mathbf{I_2}\right) = 0$$

(b)

Coil voltages:

$$\mathbf{V_1} = j16\,\mathbf{I_1} + j8\,\mathbf{I_2}$$
$$\mathbf{V_2} = j12\,\mathbf{I_2} + j8\,\mathbf{I_1}$$

Mesh equations:
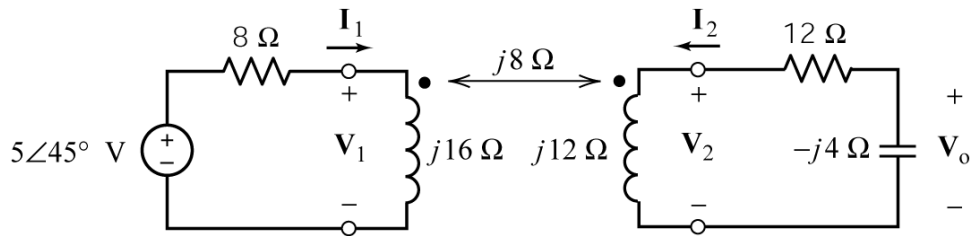
$$8\,\mathbf{I_1} + \mathbf{V_1} - 5\angle 45° = 0$$
$$-(12 - j4)\,\mathbf{I_2} - \mathbf{V_2} = 0$$

Substitute the coil voltages into the mesh equations and do some algebra:

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$$8\,\mathbf{I_1} + j16\,\mathbf{I_1} + j8\,\mathbf{I_2} - 5\angle 45° = 0$$
$$-(12 - j4)\,\mathbf{I_2} - j12\,\mathbf{I_2} - j8\,\mathbf{I_1} = 0$$

$$(8 + j16)\,\mathbf{I_1} + j8\,\mathbf{I_2} = 5\angle 45°$$
$$-j8\,\mathbf{I_1} - (12 + j8)\,\mathbf{I_2} = 0$$

$$\begin{bmatrix} 8 + j16 & j8 \\ -j8 & -12 - j8 \end{bmatrix} \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \end{bmatrix} = \begin{bmatrix} 5\angle 45° \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \end{bmatrix} = \begin{bmatrix} 0.2795\angle -4.2° \\ 0.155\angle -127.9° \end{bmatrix}$$

$$\mathbf{V_o} = -(-j4\,\mathbf{I_2}) = 0.62\angle 37.9°$$

(c)

Coil voltages:

$$\mathbf{V_1} = j16\,\mathbf{I_1} - j8\,\mathbf{I_2}$$
$$\mathbf{V_2} = j12\,\mathbf{I_2} - j8\,\mathbf{I_1}$$

Mesh equations:

$$8\,\mathbf{I_1} + \mathbf{V_1} - 5\angle 45° = 0$$
$$-(12 - j4)\,\mathbf{I_2} - \mathbf{V_2} = 0$$

Substitute the coil voltages into the mesh equations and do some algebra:

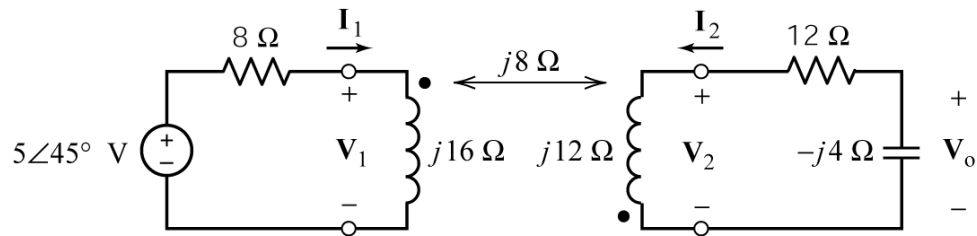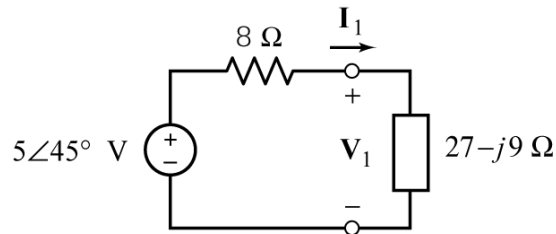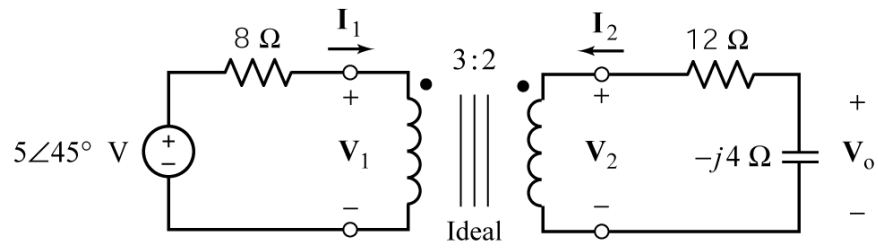$$8\,\mathbf{I_1} + j16\,\mathbf{I_1} - j8\,\mathbf{I_2} - 5\angle 45° = 0$$
$$-(12 - j4)\,\mathbf{I_2} - j12\,\mathbf{I_2} + j8\,\mathbf{I_1} = 0$$

$$(8 + j16)\,\mathbf{I_1} - j8\,\mathbf{I_2} = 5\angle 45°$$
$$j8\,\mathbf{I_1} - (12 + j8)\,\mathbf{I_2} = 0$$

$$\begin{bmatrix} 8 + j16 & -j8 \\ j8 & -12 - j8 \end{bmatrix} \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \end{bmatrix} = \begin{bmatrix} 5\angle 45° \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \end{bmatrix} = \begin{bmatrix} 0.2795\angle -4.2° \\ 0.155\angle 52.3° \end{bmatrix}$$

$$\mathbf{V_o} = -(-j4\,\mathbf{I_2}) = 0.62\angle 142.3°$$

(d)

$$\mathbf{I_1} = \frac{5\angle 45°}{8+27-j9} = \frac{5\angle 45°}{35-j9} = \frac{5\angle 45°}{36.1\angle -14.4°} = 0.1385\angle 59.4° \ \mathbf{A}$$

$$\mathbf{V_1} = \left(27-j9\right)\mathbf{I_1} = \left(27-j9\right)\left(0.1385\angle 59.4°\right) = \left(2.85\angle -18.4°\right)\left(0.1385\angle 59.4°\right) = 3.95\angle 41° \ \mathbf{V}$$

$$\mathbf{I_2} = -\frac{3}{2}\mathbf{I_1} = \left(1.5\angle -180°\right)\left(0.1385\angle 59.4°\right) = 0.02078\angle -120.6° \ \mathbf{A}$$

$$\mathbf{V_o} = -\left(-j4\,\mathbf{I_2}\right) = 0.835\angle 30.6°$$