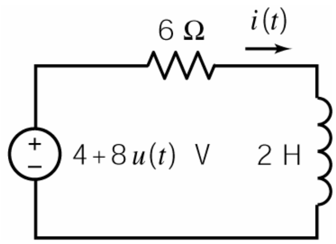
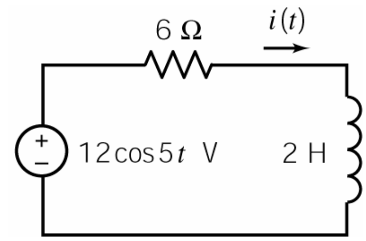
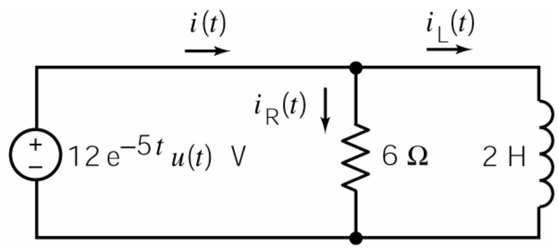
Example: The input to each of the circuits shown in Figure 10-N1 is the voltage source voltage. The output of each circuit is the current $i(t)$. Determine the output of each of the circuits.



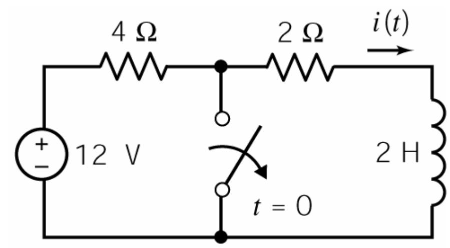
(a)



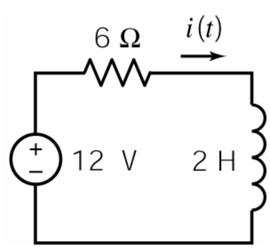
(b)



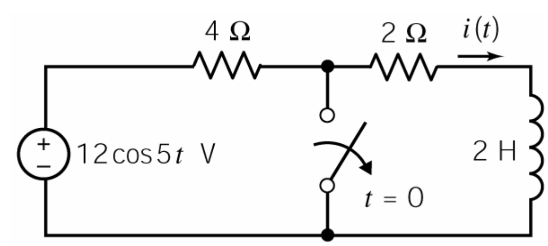
(c)



(d)



(e)



(f)

Figure 10-N1

Solution:

Case (a)

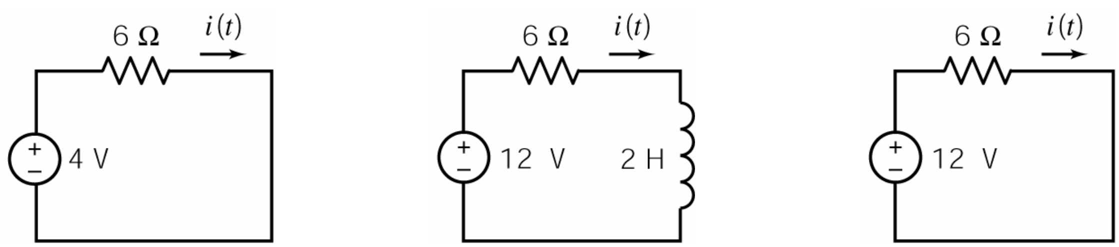
This circuit will be at steady state until time $t = 0$. Because the input is constant before time $t = 0$, all of the element voltages and currents will be constant. At time $t = 0$, the input changes abruptly, disturbing the steady state. Eventually the disturbance dies out and the circuit is again at steady state. All of the element voltages and currents will again be constant, but they will have different constant values, because the input has changed.

The three stages can be illustrated as shown in Figure 10-N2. Figure 10-N2a represents the circuit for $t < 0$. The source voltage is constant and the circuit is at steady state so the inductor acts like a short circuit. The inductor current is

$$i(t) = \frac{4}{6} = \frac{2}{3} \text{ A}$$

In particular, immediately before t = 0, $i(0-) = 0.667 \text{ A}$. The current in an inductor is continuous, so

$$i(0+) = i(0-) = 0.667 \text{ A}$$



(a) (b) (c)

Figure 10-N2

Figure 10-N2b represents the circuit immediately after t = 0. The input is constant but the circuit is not at steady state, so the inductor does not act like a short circuit. The part of the circuit that is connected to the inductor has the form of a Thevenin equivalent circuit, so we recognize that

$$R_t = 6 \ \Omega \text{ and } v_{oc} = 12 \text{ V}$$

Consequently

$$i_{sc} = \frac{12}{6} = 2 \text{ A}.$$

The time constant of the circuit is

$$\tau = \frac{L}{R_t} = \frac{2}{6} = \frac{1}{3}$$

Finally,

$$i(t) = i_{sc} + \left(i(0+) - i_{sc}\right)e^{-t/\tau} = 2 + (0.667 - 2)e^{-3t} = 2 - 1.33e^{-3t} \text{ A}$$

As t increases, the exponential part of $i(t)$ gets smaller. When $t = 5\tau = 1.667$ seconds,

$$i(t) = 2 - 1.33e^{-3(1.667)} = 2 - 0.009 \approx 2 \text{ A}$$

The exponential part of $i(t)$ has become negligible so we recognize that the circuit is again at steady state and that that new steady state current is $i(t) = 2 \text{ A}$.

Figure 10-N2c represents the circuit after the disturbance has died out and the circuit has reached steady state, that is, when $t > 5\tau$. The source voltage is constant and the circuit is at steady state so the inductor acts like a short circuit. As expected, the inductor current is 2 A.

Case (b)
This circuit does not contain a switch and the input does not change abruptly, so we expect the circuit to be at steady state. The input is sinusoidal at a frequency of 5 rad/s so all of the element currents and voltages will be sinusoidal at a frequency of 5 rad/s. We can find the steady response by representing the circuit in the frequency domain using impedances and phasors.

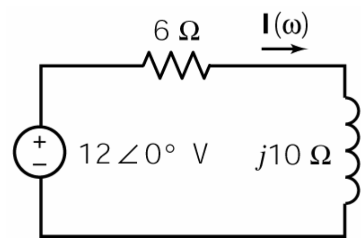


Figure 10-N3

Figure 10-N3 shows the frequency domain representation of the circuit. Ohm's law gives

$$\mathbf{I}(\omega) = \frac{12\angle 0°}{6 + j10} = \frac{12\angle 0°}{11.66\angle 59°} = 1.03\angle -59° \text{ A}$$

The corresponding current in the time domain, is

$$i(t) = 1.03\cos(5t - 59°) \text{ A}$$

Case (c)
The voltage source, resistor and inductor in this circuit are connected in parallel. The element voltage of the resistor and inductor are each equal to the voltage source voltage. The current in the resistor is given by Ohm's law to be

$$i_{\text{R}}(t) = \frac{12e^{-5t}}{6} = 2e^{-5t} \text{ A}$$

The current in the inductor is

$$i_{\text{L}}(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i_{\text{L}}(0) = \frac{1}{2}\int_0^t 12e^{-5\tau}\,d\tau + i_{\text{L}}(0)$$
$$= \frac{12}{2(-5)}\left(e^{-5t} - 1\right) + i_{\text{L}}(0) = -1.2e^{-5t} + 1.2 + i_{\text{L}}(0)$$

Finally, using KCL gives

$$i(t) = i_R(t) + i_L(t) = 2e^{-5t} - 1.2e^{-5t} + 1.2 + i_L(0) = 0.8e^{-5t} + 1.2 + i_L(0)$$

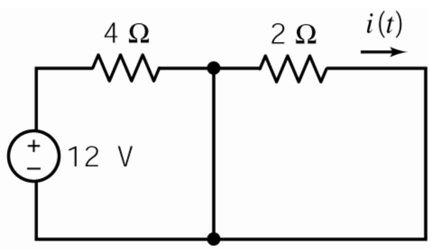
Case (d)
This circuit will be at steady state until the switch opens at time $t = 0$. Because the source voltage is constant, all of the element voltages and currents will be constant. At time $t = 0$, the switch opens, disturbing the steady state. Eventually the disturbance dies out and the circuit is again at steady state. All of the element voltages and currents will be constant, but they will have different constant values, because the circuit has changed.

The three stages can be illustrated as shown in Figure 10-N4. Figure 10-N4a represents the circuit for $t < 0$. The closed switch is represented as a short circuit. The source voltage is constant and the circuit is at steady state so the inductor acts like a short circuit. The inductor current is

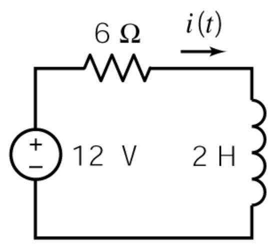
$$i(t) = 0 \text{ A}$$

In particular, immediately before $t = 0$, $i(0-) = 0 \text{ A}$. The current in an inductor is continuous, so

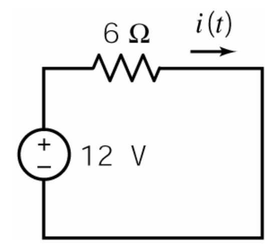
$$i(0+) = i(0-) = 0 \text{ A}$$



(a)



(b)



(c)

Figure 10-N4

Figure 10-N4b represents the circuit immediately after $t = 0$. The input is constant but the circuit is not at steady state, so the inductor does not act like a short circuit. The part of the circuit that is connected to the inductor has the form of a Thevenin equivalent circuit, so we recognize that

$$R_t = 6\ \Omega \text{ and } v_{oc} = 12 \text{ V}$$

Consequently

$$i_{sc} = \frac{12}{6} = 2 \text{ A}.$$

The time constant of the circuit is

$$\tau = \frac{L}{R_t} = \frac{2}{6} = \frac{1}{3}$$

Finally,

$$i(t) = i_{sc} + \left(i(0+) - i_{sc}\right)e^{-t/\tau} = 2 + (0-2)e^{-3t} = 2 - 2e^{-3t} \text{ A}$$

As *t* increases, the exponential part of $i(t)$ gets smaller. When $t = 5\tau = 1.667$ seconds,

$$i(t) = 2 - 2e^{-3(1.667)} = 2 - 0.013 \approx 2 \text{ A}$$

The exponential part of $i(t)$ has become negligible so we recognize that the circuit is again at steady state and that the steady state current is $i(t) = 2$ A .

Figure 10-N4c represents the circuit after the disturbance has died out and the circuit has reached steady state, that is, when $t > 5\tau$. The source voltage is constant and the circuit is at steady state so the inductor acts like a short circuit. As expected, the inductor current is 2 A.

Case (e)

This circuit does not contain a switch and the input does not change abruptly, so we expect the circuit to be at steady state. Because the source voltage is constant, all of the element voltages and currents will be constant. Because the source voltage is constant and the circuit is at steady state, the inductor acts like a short circuit. (We've encountered this circuit twice before in this example, after the disturbance died out in cases b and d.) The current is given by

$$i(t) = \frac{12}{6} = 2 \text{ A}$$

Case (f)

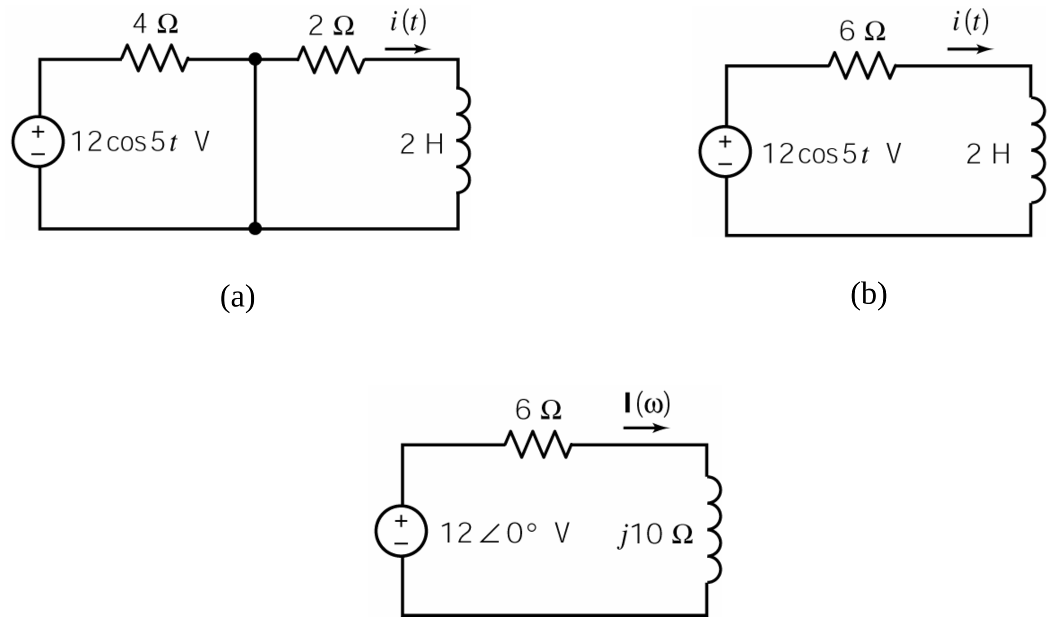
We expect that this circuit will be at steady state before the switch opens. As before, opening the switch will change the circuit and disturb the steady state. Eventually the disturbance will die out and the circuit will again be at steady state. We will see that the steady state current is constant before the switch opens and sinusoidal after the switch opens.

Figure 10N5a shows the circuit before the switch opens. Applying KVL gives

$$2\,i(t) + 2\frac{d}{dt}i(t) = 0$$

Consequently, the inductor current is $i(t) = 0$ before the switch opens. The current in an inductor is continuous, so

$$i(0+) = i(0-) = 0 \text{ A}$$



(c)

Figure 10-N5

Figure 10-N5b represents the circuit after the switch opens. We can determine the inductor current by adding the natural response to forced response and then using the initial condition to evaluate the constant in the natural response.

First, we find the natural response. The part of the circuit that is connected to the inductor has the form of the Thevenin equivalent circuit, so we recognize that

$$R_\mathrm{t} = 6\ \Omega$$

The time constant of the circuit is

$$\tau = \frac{L}{R_\mathrm{t}} = \frac{2}{6} = \frac{1}{3}$$

The natural response is

$$i_\mathrm{n}(t) = K\,e^{-3t}\ \text{A}$$

We can use the steady state response as the forced response. As in case b, we obtain the steady state response by representing the circuit in the frequency as shown in Figure 10N-5c. As before, we find $\mathbf{I}(\omega) = 1.03\angle -59°\ \text{A}$. The forced response is

$$i_\mathrm{f}(t) = 1.03\cos(5t - 59°)\ \ \text{A}$$

Then

$$i(t) = i_\mathrm{n}(t) + i_\mathrm{f}(t) = K\,e^{-3t} + 1.03\cos(5t - 59°)\ \text{A}.$$

At $t = 0$,

$$i(0) = K\,e^{-0} + 1.03\cos(-59°) \;\; = K + 0.53$$

so

$$i(t) = -0.53e^{-3t} + 1.03\cos(5t - 59°)\ \text{A}$$