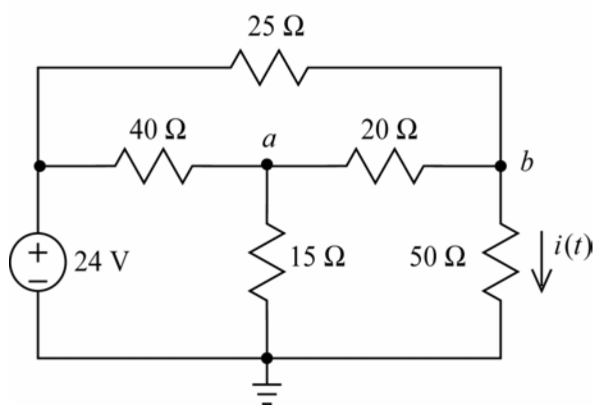
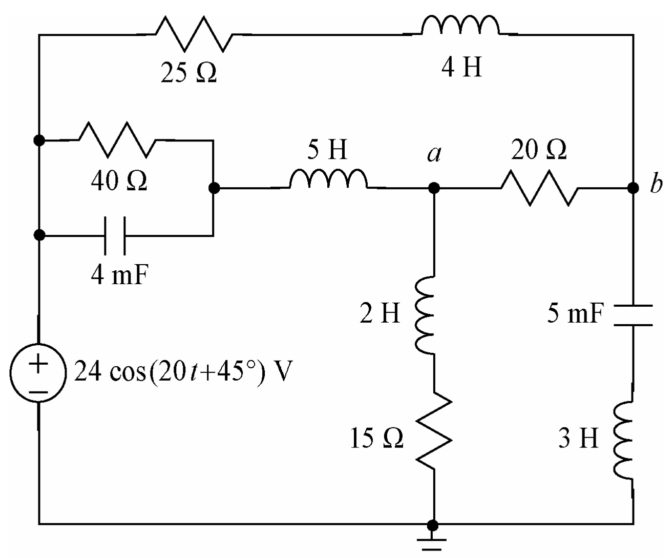
Example1 : Determine the node voltages at nodes *a* and *b* of each of the circuits shown in Figure 1.



(*a*)



(*b*)

Figure 1

Solution:

(a) The node equations are

$$\frac{24-v_a}{40}=\frac{v_a-v_b}{20}+\frac{v_a}{15}$$

$$\frac{24-v_b}{25}+\frac{v_a-v_b}{20}=\frac{v_b}{50}$$

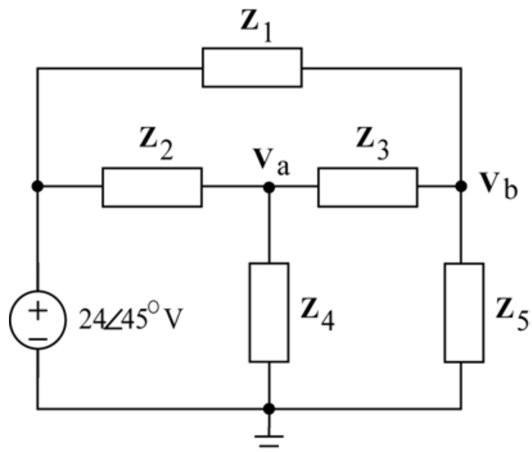
or

$$\begin{bmatrix} \frac{1}{40}+\frac{1}{20}+\frac{1}{15} & -\frac{1}{20} \\ -\frac{1}{20} & \frac{1}{25}+\frac{1}{20}+\frac{1}{50} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} \frac{24}{40} \\ \frac{24}{25} \end{bmatrix}$$

Solving using MATLAB gives

$$v_a = 8.713 \text{ V} \quad \text{and} \quad v_b = 12.69 \text{ V}$$

(b) Use phasors and impedances to represent the circuit in the frequency domain as



where

$$\mathbf{Z}_1 = 25 + j(20)4 = 25 + j80 = 83.82\angle 72.7° \ \Omega$$

$$\mathbf{Z}_2 = \left(40 \,||\, \frac{1}{j(20)(0.004)}\right) + j(20)5 = 3.56 + j88.6 = 88.68\angle 87.7° \ \Omega$$

$$\mathbf{Z}_3 = 20 \ \Omega$$

$$\mathbf{Z}_4 = 15 + j(20)2 = 15 + j40 = 42.72\angle 69.4°$$

$$\mathbf{Z}_5 = j(20)3 + \frac{1}{j(20)(0.005)} = j50 = 50\angle 90° \ \Omega$$

The node equations are

$$\frac{24\angle 45° - \mathbf{V}_a}{\mathbf{Z}_2} = \frac{\mathbf{V}_a}{\mathbf{Z}_4} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3}$$

$$\frac{24\angle 45° - \mathbf{V}_b}{\mathbf{Z}_1} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3} = \frac{\mathbf{V}_b}{\mathbf{Z}_5}$$

$$\begin{bmatrix} \frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4} & -\frac{1}{\mathbf{Z}_3} \\ -\frac{1}{\mathbf{Z}_3} & \frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_5} \end{bmatrix} \begin{bmatrix} \mathbf{V}_\text{a} \\ \mathbf{V}_\text{b} \end{bmatrix} = \begin{bmatrix} \frac{24\angle 45°}{\mathbf{Z}_2} \\ \frac{24\angle 45°}{\mathbf{Z}_1} \end{bmatrix}$$

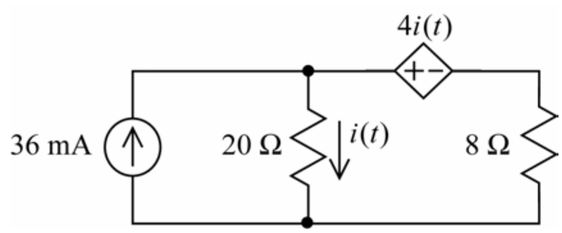
Solving using MATLAB gives

$$\mathbf{V}_\text{a} = 7.89\angle 44.0°$$
$$\mathbf{V}_\text{b} = 8.45\angle 45.1°$$

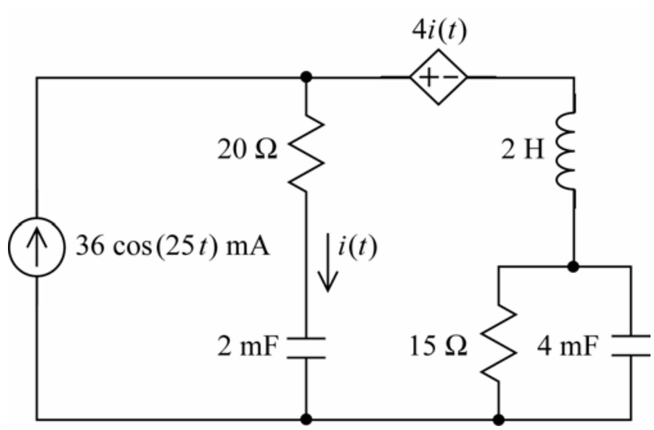
so

$$v_\text{a}(t) = 7.89\cos(20t+44°)\ \text{V}$$
$$v_\text{a}(t) = 8.45\cos(20t+45.1)\ \text{V}$$

Example 2 Determine the steady state current, $i(t)$, for each of the circuits shown in 2.



(*a*)



(*b*)

Figure 2

Solution

(a) Use KVL to see that the voltage across the 8 Ω resistor is $20i(t)-4i(t)=16i(t)$.

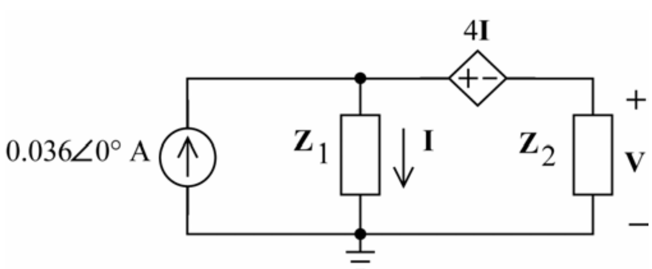
Apply KCL to the supernode corresponding to the dependent voltage source to get

$$0.036=i(t)+\frac{16i(t)}{8}=3i(t)$$

so

$$i(t)=12\text{ mA}$$

(b) Represent the circuit in the frequency domain using phasors and impedances.



Where

$$\mathbf{Z}_1=20+\frac{1}{j(25)(0.002)}=20-j20\ \Omega$$

$$\mathbf{Z}_2=j50+\left(15\,||\,\frac{1}{j(25)(0.004)}\right)=43.3\angle 83.9°\ \Omega$$

Use KVL to get

$$\mathbf{V}=\mathbf{Z}_1\mathbf{I}-4\mathbf{I}=(\mathbf{Z}_1-4)\mathbf{I}$$

Then apply KCL to the supernode corresponding to the dependent source to get

$$0.036\angle 0°=\mathbf{I}+\frac{(\mathbf{Z}_1-4)\mathbf{I}}{\mathbf{Z}_2}=\left(\frac{\mathbf{Z}_1+\mathbf{Z}_2-4}{\mathbf{Z}_2}\right)\mathbf{I}$$

so

$$\mathbf{I}=\frac{\mathbf{Z}_2(0.036\angle 0°)}{\mathbf{Z}_1+\mathbf{Z}_2-4}=50.4\angle 35.7°\text{ mA}$$

so

$$i(t)=50.4\cos(25t+35.7°)\text{ mA}$$