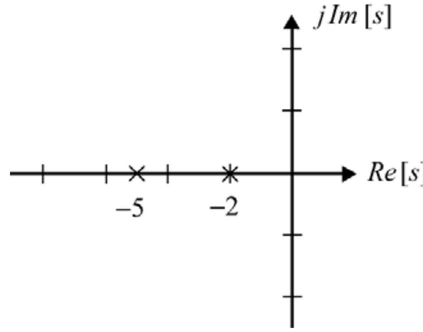
Example

The input to a linear circuit is the voltage, v_{i}. The output is the voltage, v_{o}. The transfer function of the circuit is

$$H(s) = \frac{V_{\mathrm{o}}(s)}{V_{\mathrm{i}}(s)}$$



The poles and zeros of $H(s)$ are shown on this pole-zero diagram. (There are no zeros.) The dc gain of the circuit is

$$\mathbf{H}(0) = 5$$

Determine the step response of the circuit.

Solution:

The transfer function of the circuit is

$$H(s) = \frac{a}{(s+2)(s+5)}$$

where a is a constant to be determined. The circuit is stable because all its poles lie in the right half of the s-plane. Consequently,

$$\mathbf{H}(\omega) = H(s)\big|_{s=j\omega} = \frac{a}{(2+j\omega)(5+j\omega)} = \frac{\frac{a}{10}}{\left(1+j\frac{\omega}{2}\right)\left(1+j\frac{\omega}{5}\right)}$$

At dc ($\omega = 0$)

$$5 = \mathbf{H}(0) = \frac{a}{10} \quad \Rightarrow \quad a = 50$$

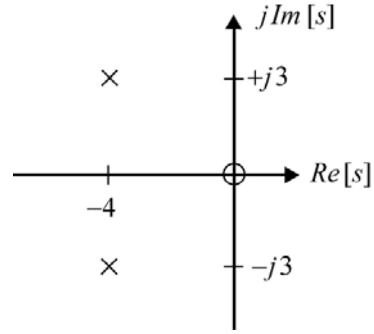
The step response is given by

$$v_{\mathrm{o}}(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{50}{s(s+2)(s+5)}\right] = \mathcal{L}^{-1}\left[\frac{5}{s} + \frac{\frac{10}{3}}{s+5} - \frac{\frac{25}{3}}{s+2}\right] = \left(5 + \frac{10}{3}e^{-5t} - \frac{25}{3}e^{-2t}\right)u(t) \ \text{ V}$$

Example

The input to a linear circuit is the voltage, v_{i}. The output is the voltage, v_{o}. The transfer function of the circuit is

$$H(s) = \frac{V_{\text{o}}(s)}{V_{\text{i}}(s)}$$



The poles and zeros of $H(s)$ are shown on this pole-zero diagram. At $\omega = 5$ rad/s the gain of the circuit is

$$\mathbf{H}(5) = 10$$

Determine the step response of the circuit.

Solution:

The transfer function of the circuit is

$$H(s) = \frac{a(s-0)}{(s+4-j3)(s+4+j3)} = \frac{a\,s}{s^2+8s+25}$$

where *a* is a constant to be determined. The circuit is stable because all its poles lie in the right half of the *s*-plane. Consequently,

$$\mathbf{H}(\omega) = H(s)\big|_{s=j\omega} = \frac{j\,a\,\omega}{(j\omega)^2 + j8\omega + 25} = \frac{j\,a\,\omega}{(25-\omega^2) + j8\omega}$$

At $\omega = 5$ rad/s

$$10 = \frac{j\,a\,5}{(25-5^2) + j8(5)} = \frac{a}{8} \quad \Rightarrow \quad a = 80$$

The step response is given by

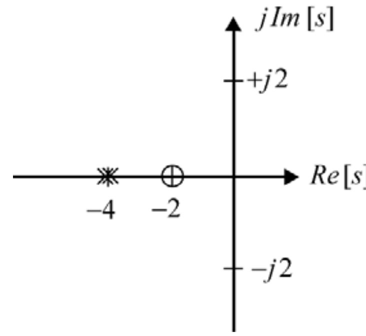
$$v_{\text{o}}(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{80\,s}{s(s^2+8s+25)}\right] = \mathcal{L}^{-1}\left[\frac{80}{s^2+8s+25}\right] = \mathcal{L}^{-1}\left[\frac{80}{3}\times\frac{3}{(s+4)^2+3^2}\right]$$

$$= \frac{80}{3}e^{-4t}\sin(4t)u(t) \text{ V}$$

Example

The input to a linear circuit is the voltage, v_i. The output is the voltage, v_o. The transfer function of the circuit is

$$H(s) = \frac{V_\mathrm{o}(s)}{V_\mathrm{i}(s)}$$



The poles and zeros of $H(s)$ are shown on this pole-zero diagram. (There is a double pole at s = – 4.) The dc gain of the circuit is

$$\mathbf{H}(0) = 5$$

Determine the step response of the circuit.

Solution:

The transfer function of the circuit is

$$H(s) = \frac{a(s+2)}{(s+4)^2}$$

where *a* is a constant to be determined. The circuit is stable because all its poles lie in the right half of the *s*-plane. Consequently,

$$\mathbf{H}(\omega) = H(s)\big|_{s=j\omega} = \frac{a(j\omega+2)}{(j\omega+4)^2} = \frac{\frac{a}{8}\left(1+j\frac{\omega}{2}\right)}{\left(1+j\frac{\omega}{4}\right)^2}$$

At dc (ω = 0)

$$5 = \mathbf{H}(0) = \frac{a}{8} \quad \Rightarrow \quad a = 40$$

The step response is given by

$$v_\mathrm{o}(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{40(s+2)}{s(s+4)^2}\right] = \mathcal{L}^{-1}\left[\frac{5}{s} - \frac{5}{s+4} + \frac{20}{(s+4)^2}\right] = \left(5 + (20t-5)e^{-4t}\right)u(t) \ \text{ V}$$

Example:
The input to a circuit is the voltage, v_i. The step response of the circuit is

$$v_o = 5e^{-4t}\sin(2t)u(t) \ \text{V}$$

Sketch the pole-zero diagram for this circuit.

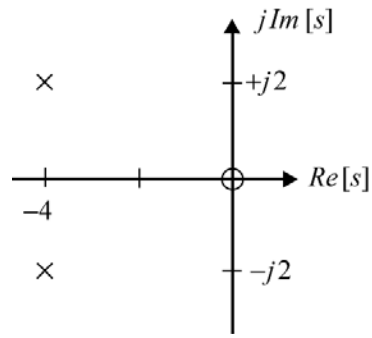
Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[5e^{-4t}\sin(2t)u(t)\right] = 5\times\left.\frac{2}{s^2+2^2}\right|_{s\leftarrow s+4} = \frac{10}{(s+4)^2+2^2}$$

$$= \frac{10}{s^2+8s+20} = \frac{10}{(s+4-j2)(s+4+j2)}$$

Consequently

$$H(s) = \frac{10\,s}{(s+4-j2)(s+4+j2)}$$



Example:
The input to a circuit is the voltage, v_i. The step response of the circuit is

$$v_o = 5t\,e^{-4t}u(t) \ \text{V}$$

Sketch the pole-zero diagram for this circuit.

Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[5t\,e^{-4t}u(t) \ \text{V}\right] = 5\times\left.\frac{1}{s^2}\right|_{s\leftarrow s+4} = \frac{5}{(s+4)^2}$$

Consequently

$$H(s) = \frac{5\,s}{(s+4)^2}$$