**Example**

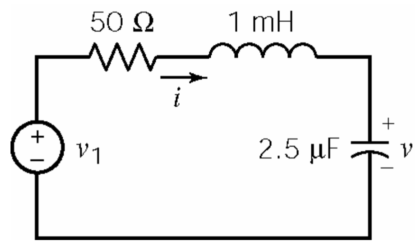
The input to this circuit is the voltage source voltage

$$v_1 = 2e^{-2\times10^4 t} \text{ V}$$

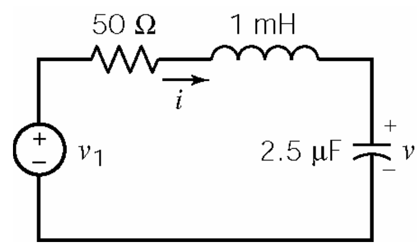
The output is the current, *i*. The initial conditions are

$$i(0) = 1 \text{ A}, \; v(0) = 8 \text{ V}$$

Determine the current *i* for $t \ge 0$.




**Solution:**




$$v_1 = 2e^{-2\times10^4 t} \text{ V}, \quad i(0) = 1 \text{ A}, \; v(0) = 8 \text{ V}$$

KVL:

$$50i + 0.001\frac{di}{dt} + v = 2e^{-2\times10^4 t} \quad \text{for } t \ge 0$$

The capacitor current and voltage are related by

$$i = \left(2.5\times10^{-6}\right)\frac{dv}{dt}$$

Taking the Laplace transforms of these equations:

$$50\,I(s) + 0.001\left[s\,I(s) - i(0)\right] + V(s) = \frac{2}{s + 2\times10^4}$$

$$I(s) = \left(2.5\times10^{-6}\right)\left[sV(s) - v(0)\right]$$

Solving for I(s) yields

$$I(s) = \frac{s^2 + 1.4\times10^4 s - 1.6\times10^8}{\left(s+10^4\right)\left(s+2\times10^4\right)\left(s+4\times10^4\right)} = \frac{A}{s+10^4} + \frac{B}{s+2\times10^4} + \frac{C}{s+4\times10^4}$$

where

$$A = \left(s+10^4\right)I(s)\Big|_{s=-10^4} = \left.\frac{s^2 + 1.4\times10^4 s - 1.6\times10^8}{\left(s+2\times10^4\right)\left(s+4\times10^4\right)}\right|_{s=-10^4} = \frac{-2\times10^8}{3\times10^8} = \frac{-2}{3}$$

$$B = \left(s+2\times10^4\right)I(s)\Big|_{s=-2\times10^4} = \left.\frac{s^2 + 1.4\times10^4 s - 1.6\times10^8}{\left(s+10^4\right)\left(s+4\times10^4\right)}\right|_{s=-2\times10^4} = \frac{0.4\times10^8}{2\times10^8} = \frac{1}{5}$$

$$C = \left(s+4\times10^4\right)I(s)\Big|_{s=-4\times10^4} = \left.\frac{s^2 + 1.4\times10^4 s - 1.6\times10^8}{\left(s+10^4\right)\left(s+2\times10^4\right)}\right|_{s=-4\times10^4} = \frac{8.8\times10^8}{6\times10^8} = \frac{22}{15}$$

Finally

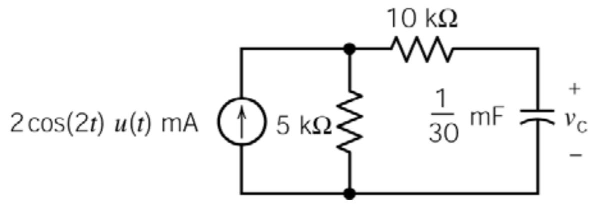
$$I(s) = -\frac{2/3}{s+10000} + \frac{1/5}{s+20000} + \frac{22/15}{s+40000} \quad \Rightarrow \quad i(t) = \frac{1}{15}\left[-10e^{-10000t} + 3e^{-20000t} + 22e^{-40000t}\right]u(t) \text{ A}$$

**Example**

Determine the capacitor voltage, $v_c$, for $t \ge 0$.

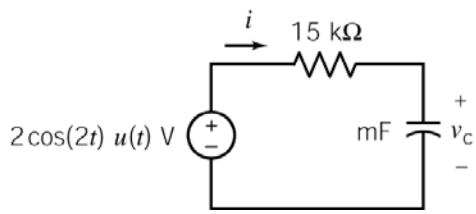
The initial conditions is

$$v_C(0) = 0$$




**Solution:**

After a source transformation we have




Then

$$\left.\begin{aligned} v_c + 15\times 10^3\, i &= 10\cos 2t \\ i &= \left(\frac{1}{30}\times 10^{-3}\right)\frac{d\,v_c}{dt} \end{aligned}\right\} \quad \Rightarrow \quad \frac{d\,v_c}{dt} + 2\,v_c = 20\cos 2t$$

Taking the Laplace Transform yields:

$$sV_C(s) - v_C(0) + 2V_C(s) = 20\,\frac{s}{s^2+4}$$

With $v_C(0) = 0$, we have

$$V_C(s) = \frac{20s}{(s+2)(s^2+4)} = \frac{A}{s+2} + \frac{B}{s+j2} + \frac{B^*}{s-j2}$$

where

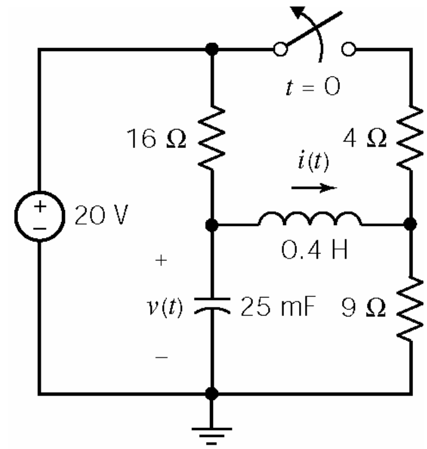
$$A = \left.\frac{20s}{s^2+4}\right|_{s=-2} = \frac{-40}{8} = -5,\quad B = \left.\frac{20s}{(s+2)(s-j\,2)}\right|_{s=-j2} = \frac{5}{1-j} = \frac{5}{2} + j\,\frac{5}{2} \quad \text{and} \quad B^* = \frac{5}{2} - j\,\frac{5}{2}$$

Then

$$V_C(s) = \frac{-5}{s+2} + \frac{\frac{5}{2}+j\frac{5}{2}}{s+j2} + \frac{\frac{5}{2}-j\frac{5}{2}}{s-j2} \quad \Rightarrow \quad v_C(t) = -5e^{-2t} + 5(\cos 2t + \sin 2t)\ \text{V}$$

**Example:**

Determine the inductor current, $i(t)$, in this circuit.



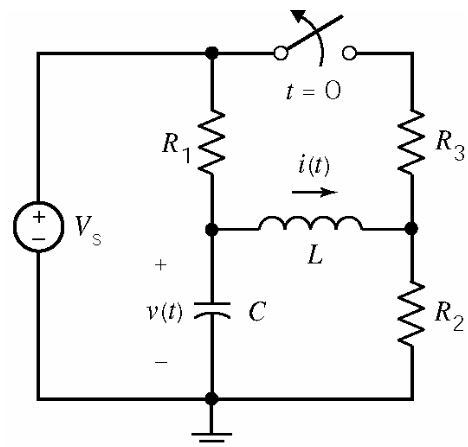

**Solution**

After the switch opens, apply KCL and KVL to get

$$R_1\left(i(t)+C\frac{d}{dt}v(t)\right)+v(t)=V_s$$

Apply KVL to get

$$v(t)=L\frac{d}{dt}i(t)+R_2\,i(t)$$




Substituting $v(t)$ into the first equation gives

$$R_1\left(i(t)+C\frac{d}{dt}\left(L\frac{d}{dt}i(t)+R_2\,i(t)\right)\right)+L\frac{d}{dt}i(t)+R_2\,i(t)=V_s$$

then $$R_1\,C\,L\frac{d^2}{dt^2}i(t)+\left(R_1\,C\,R_2+L\right)\frac{d}{dt}i(t)+\left(R_1+R_2\right)i(t)=V_s$$

Dividing by $R_1\,C\,L$:

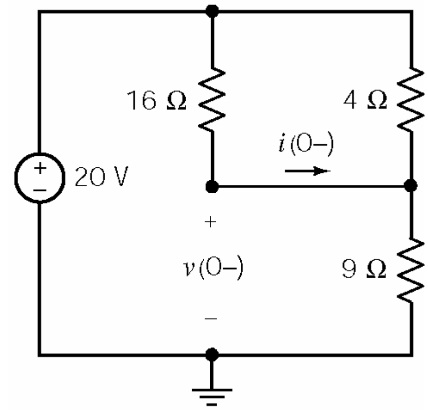
$$\frac{d^2}{dt^2}i(t)+\left(\frac{R_1\,C\,R_2+L}{R_1\,C\,L}\right)\frac{d}{dt}i(t)+\left(\frac{R_1+R_2}{R_1\,C\,L}\right)i(t)=\frac{V_s}{R_1\,C\,L}$$

With the given values: $$\frac{d^2}{dt^2}i(t)+25\frac{d}{dt}i(t)+156.25\,i(t)=125$$

Taking the Laplace transform:

$$\left[s^2\,I(s)-\left(\frac{d}{dt}i(0+)+s\,i(0+)\right)\right]+25\left[s\,I(s)-i(0+)\right]+156.25\,I(s)=\frac{125}{s}$$

We need the initial conditions. For $t < 0$, The switch is closed and the circuit is at steady state. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit. Using voltage division




$$v(0-) = \frac{9}{9 + (16 \,\|\, 4)} \, 20 = 14.754 \text{ V}$$

Then, using current division

$$i(0-) = \left(\frac{4}{16+4}\right) \frac{v(0-)}{9} = 0.328 \text{ A}$$

The capacitor voltage and inductor current are continuous so $v(0+) = v(0-)$ and $i(0+) = i(0-)$. After the switch opens

$$v(t) = L \frac{d}{dt} i(t) + R_2 \, i(t) \quad \Rightarrow \quad \frac{d}{dt} i(0+) = \frac{v(0+)}{0.4} + \frac{9\, i(0+)}{0.4} = \frac{14.754}{0.4} + \frac{9\,(0.328)}{0.4} = 29.508$$

Substituting these initial conditions into the Laplace transformed differential equation gives

$$\left[s^2 \, I(s) - (29.508 + 0.328\, s)\right] + 25\left[s\, I(s) - 0.328\right] + 156.25\, I(s) = \frac{125}{s}$$

$$\left(s^2 + 25\, s + 156.25\right) I(s) = \frac{125}{s} + (29.508 + 0.328\, s) + 25\,(0.328)$$

so

$$I(s) = \frac{0.328\, s^2 + \left(29.508 + 25\,(0.328)\right) + 125}{s\left(s^2 + 25\, s + 156.25\right)}$$

$$= \frac{0.328\, s^2 + \left(29.508 + 25\,(0.328)\right) + 125}{s\,(s + 12.5)^2} = \frac{-0.471}{s + 12.5} + \frac{23.6}{(s + 12.5)^2} + \frac{0.8}{s}$$

Taking the inverse Laplace transform

$$i(t) = 0.8 + e^{-12.5t}\,(23.6\,t - 0.471) \quad \text{A} \quad \text{for } t \ge 0$$

so

$$i(t) = \begin{cases} 0.328 \quad \text{A} \quad \text{for } t \le 0 \\ 0.8 + e^{-12.5t}\,(23.6\,t - 0.471) \quad \text{A} \quad \text{for } t \ge 0 \end{cases}$$