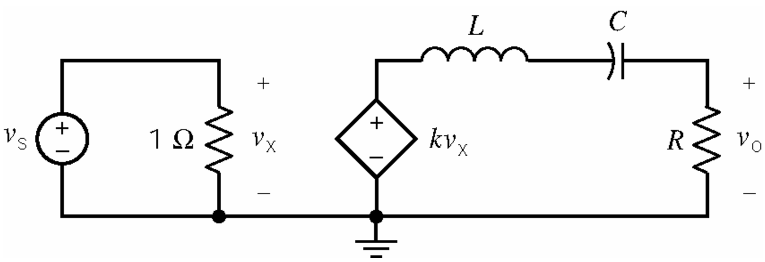
Design Problems

Example:



The input of this circuit is the voltage source voltage, v_s. The output is the resistor voltage, v_o. Design this circuit to have the step response

$$v_o = 5te^{-4t}u(t) \text{ V}$$

Solution:
Equating the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

$$V_o(s) = \frac{\frac{kR}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{5}{(s+4)^2}$$

Equating the poles:

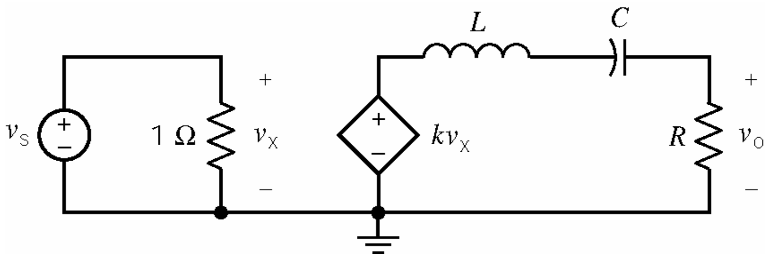
$$s_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -4 \pm j0$$

Summarizing the results of these comparisons:

$$\frac{R}{2L} = 4, \quad R = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad \frac{kR}{L} = 5$$

Pick L = 1 H, then k = 0.625 V/V, R = 8 Ω and C = 0.0625 F.

Example:



The input of this circuit is the voltage source voltage, v_s. The output is the resistor voltage, v_o. Design this circuit to have the step response

$$v_o = 5e^{-4t}\sin(2t)u(t) \text{ V}$$

Solution:
Equating the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

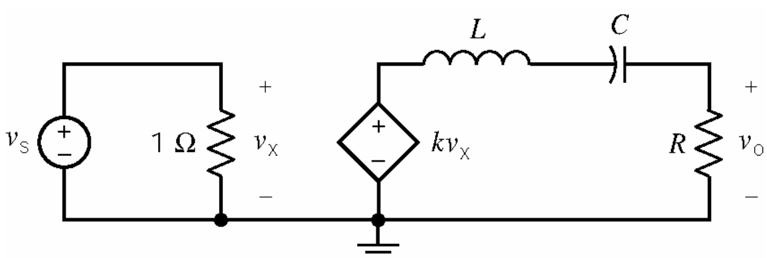
$$V_o(s) = \frac{\frac{kR}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{10}{(s+4)^2 + 4} = \frac{10}{s^2 + 8s + 20}$$

Equating coefficients:

$$\frac{R}{L} = 8, \quad \frac{1}{LC} = 20 \quad \text{and} \quad \frac{kR}{L} = 10$$

Pick L = 1 H, then k = 1.25 V/V, R = 8 Ω and C = 0.05 F.

Example:



The input of this circuit is the voltage source voltage, v_s. The output is the resistor voltage, v_o. Design this circuit to have the step response

$$v_o = 5\left(e^{-2t} - e^{-4t}\right)u(t) \text{ V}$$

Solution:
Equating the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

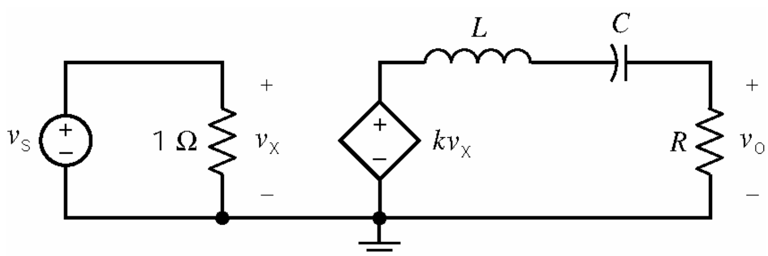
$$V_o(s) = \frac{\frac{kR}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{5}{s+2} - \frac{5}{s+4} = \frac{10}{s^2+6s+8}$$

Equating coefficients:

$$\frac{R}{L} = 6, \quad \frac{1}{LC} = 8 \quad \text{and} \quad \frac{kR}{L} = 10$$

Pick L = 1 H, then k = 1.667 V/V, R = 6 Ω and C = 0.125 F.

Example:



The input of this circuit is the voltage source voltage, v_s. The output is the resistor voltage, v_o. Design this circuit to have the step response

$$v_o = 5\left(e^{-2t} + e^{-4t}\right)u(t) \ \text{ V}$$

Solution:
Comparing the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

$$V_o(s) = \frac{\frac{kR}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \neq \frac{5}{(s+2)} + \frac{5}{(s+4)} = \frac{10s+30}{s^2+6s+8}$$

These two functions can not be made equal by any choice of k, R, C and L because the numerators have different forms.