

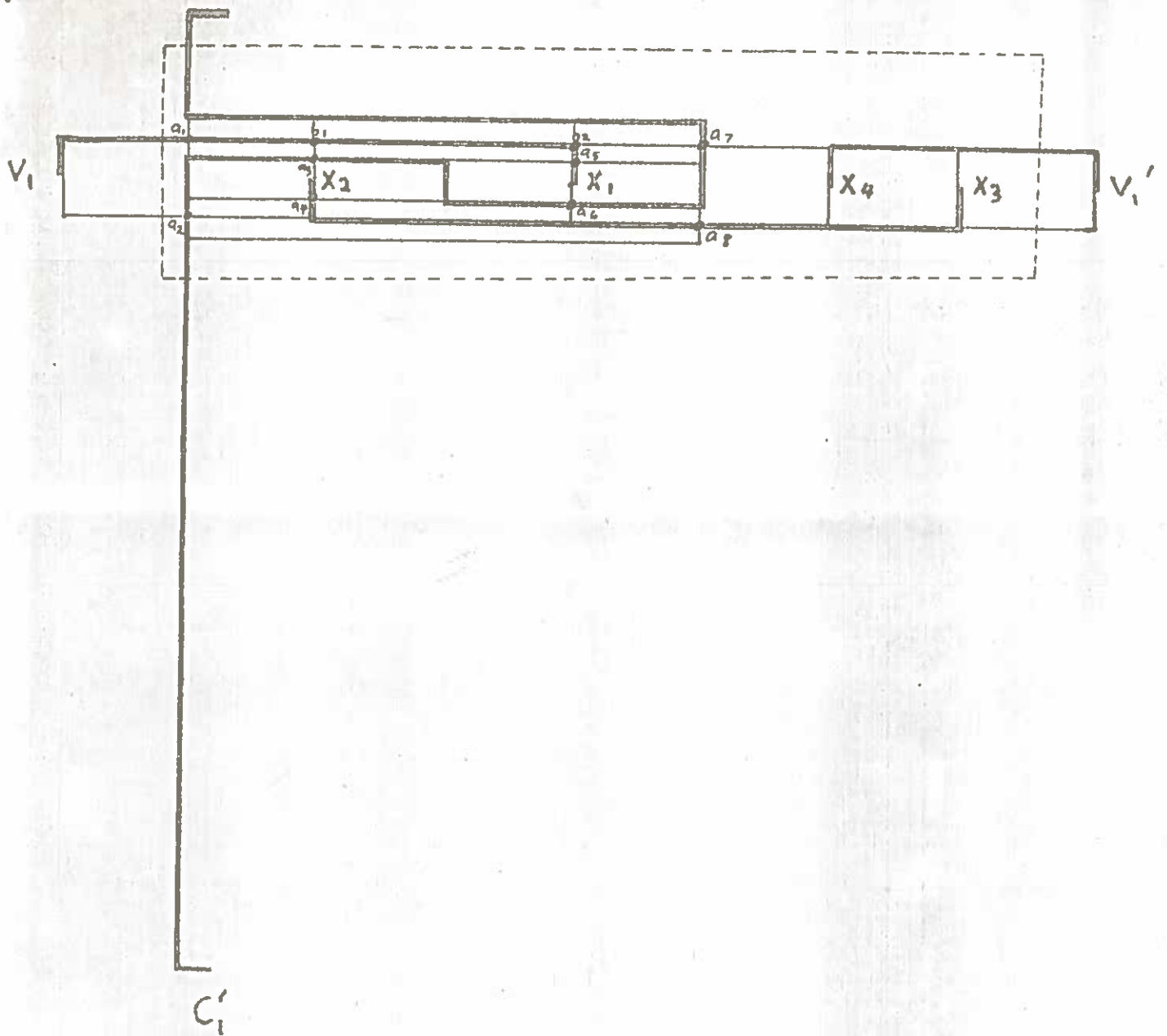


Figure 2.

Removal of nonplanarity from Figure 1. The heavy lines show the high path taken at v_1 and v_1' and the path between C_1 and C_1' .

Lemma: Let G be the graph in figure 3. Then the interconnection problem on G and sets $\{u_i, v_i\}$, $i=0, \dots, 3$, and $\{p_j, p_k\}$ is solvable if and only if $k=j+2(\text{mod } 4)$.

proof: Clear.

We will now construct a planar graph from the graph in Figure 1.

Replace the portion within the dotted line in figure 1 by the portion within the dotted line in figure 2. Further, put little "elbows" around points $a_1, \dots, a_8$ as indicated in Figure 3. (This should have been mentioned in the paper.) I claim that if the high or low path

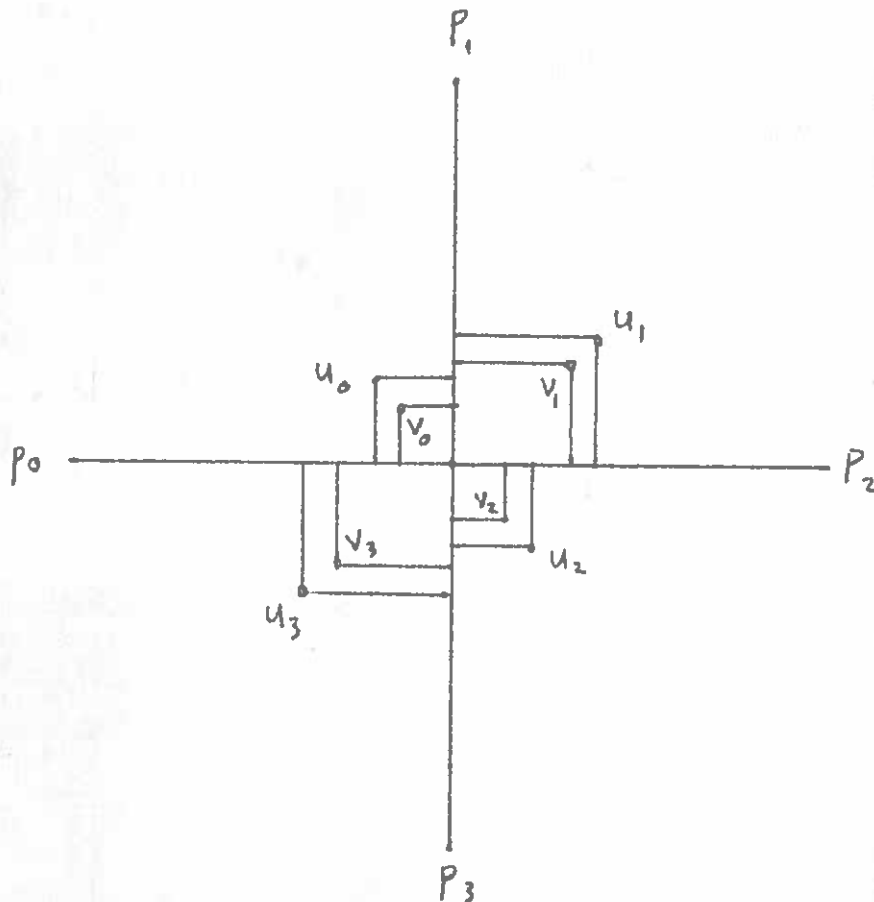


Figure 3.

is taken at v_1 , then the same type (high or low) path must be taken at v'_1 in order to connect the sets $\{v_1, x_1\}$, $\{x_2, x_3\}$, and $\{x_4, v'_1\}$. For, supposing the high path is taken at v_1 , a_1 must be part of this path. Then by the lemma, the path cannot bend up or down at a_1 but must continue on to the right. At b_1 the path cannot bend down to a_3 because (again by the lemma) it could only continue to x_2 . Thus, it goes on to b_2 , but not to a_7 because it could only continue on to the right. This forces it to bend down at b_2 to go to a_5 and x_1 . Now the path connecting x_2 and x_3 cannot go from x_2 to a_3 because it would then have to continue up to b_1 which is already part of the path from v_1 to x_1 . Therefore it goes down to a_4 and proceeds as shown in Figure 2. This forces the path from x_4 to v'_1 to be high. Similarly, if the low path is taken at v_1 , then the low path must be taken at v'_1 . Note that the path from C_1 to C'_1 is not blocked in either case.

Figure 4 shows how the other type of nonplanarity is removed. Again the same type of path must be taken at v_2 and v'_2 , but the path from C_1 to C'_1 is blocked if and only if the high path is taken.

By repeating this procedure for all pairs of clauses and variables, we obtain a planar graph with some subsets of its points which is a solvable interconnection problem if and only if the graph in Figure 1 is solvable. Clearly this procedure is polynomial in time.

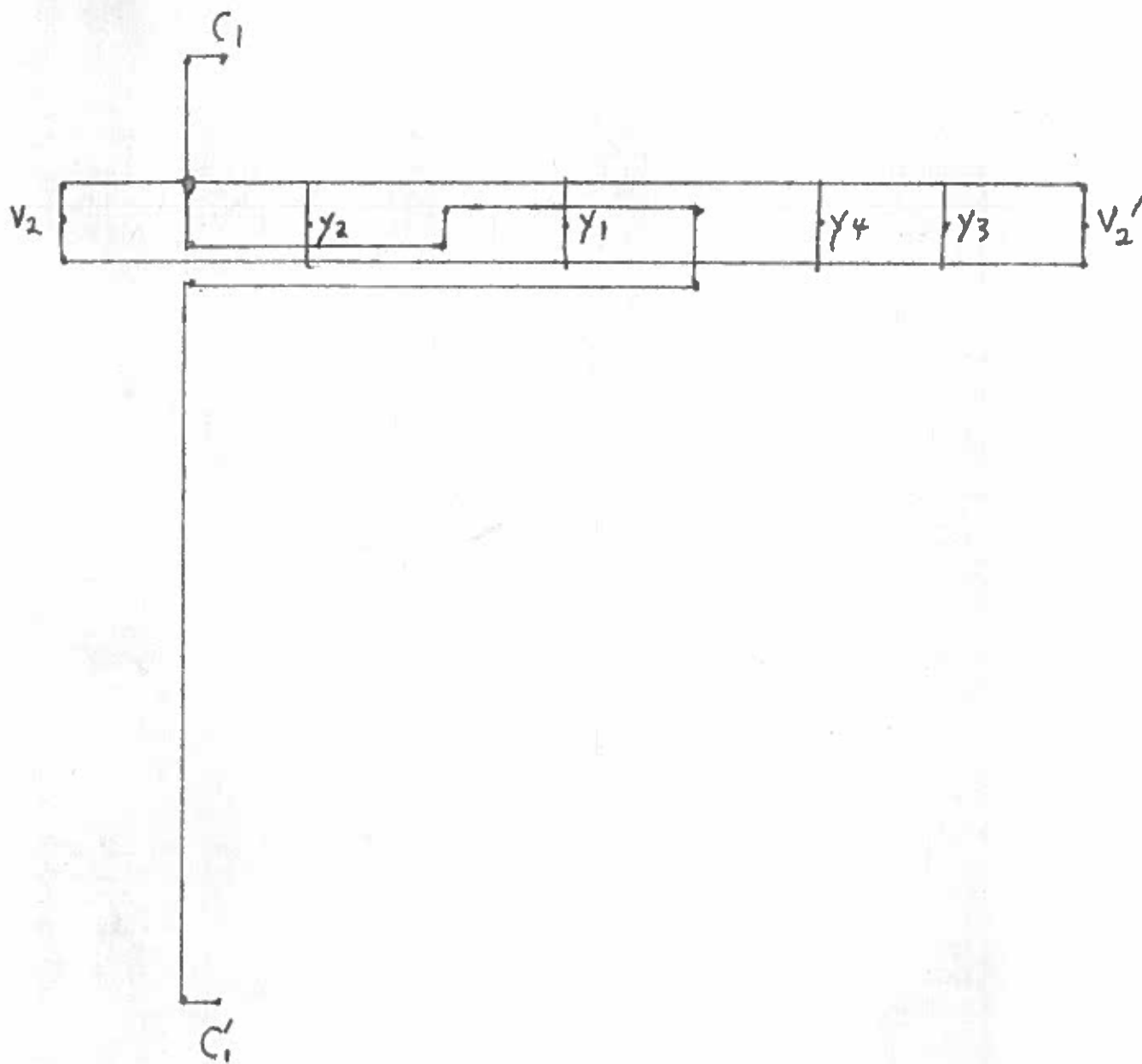


Figure 4

Regarding the "percentage of connections" problem, the exact statement is something like this. Let c be a rational constant in $(0,1]$. Given a graph G and n disjoint subsets of its points, are there at least cn of these subsets such that they may be connected by cn disjoint subgraphs? Perhaps the easiest way to prove that it is NP-complete is to reduce the standard interconnection problem to it as follows. Let G be a graph, and $S_1, \dots, S_m$ be the disjoint subsets we wish to connect. Add the isolated points $p_1, \dots, p_r$ and $q_1, \dots, q_r$ to G , where $r = \lfloor (1/c - 1)m \rfloor$. Then G and $S_1, \dots, S_m$ is a solvable interconnection problem if and only if G with $p_1, \dots, p_r$ and $q_1, \dots, q_r$ added and $S_1, \dots, S_m, \{p_1, q_1\}, \dots, \{p_r, q_r\}$ is a solvable "c-percent" interconnection problem.