

Homework 6

3.2 :

4. One way to do it is to use equations (3.2) and (3.4):

$$\phi U \psi \equiv \phi W \psi \wedge F \psi \quad (3.2)$$

$$\equiv \psi R(\phi \vee \psi) \wedge F \psi \quad (3.4)$$

Or, it can be done more directly:

Let \mathcal{M} be a model and π be a path in \mathcal{M} . First, assume $\pi \models \phi U \psi$. From Definition 3.6 parts 11. and 13.. and the fact that ϕ implies $\phi \vee \psi$, it follows that $\pi \models \psi R(\phi \vee \psi)$, and from part 10., $\pi \models F \psi$. Therefore $\pi \models \psi R(\phi \vee \psi) \wedge F \psi$.

Now assume $\pi \models \psi R(\phi \vee \psi) \wedge F \psi$. By Definition 3.6 part 10., there is some i such that $\pi^i \models \psi$. Take the first such i . There are two cases.

First case: $i = 1$. Then $\pi \models \phi U \psi$ by part 11.

Second case: $i > 1$. Then $\pi^j \models \phi$ for all $j < i$ because otherwise there is some $j < i$ such that $\pi^j \not\models \phi$. Also $\pi^j \not\models \psi$ by our choice of i . That is, $\pi^j \models \neg \phi \wedge \neg \psi$. Since $\neg \phi \wedge \neg \psi \equiv \neg(\phi \vee \psi)$, $\pi^j \not\models \phi \vee \psi$. By part 13., $\pi \not\models \psi R(\phi \vee \psi)$, contradiction. Therefore, by part 11., $\pi \models \phi U \psi$.

3.3:

1. (b) No, because ready is true in both initial states, so reqU¬busy is true.
2. Safety: There is no state where both c_1 and c_2 are true, so the system is safe.

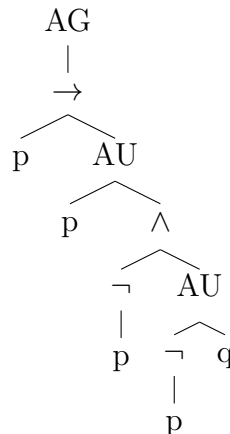
Liveness: From every state where t_1 is true, every path leads to a state where c_1 is true. Similarly for t_2 and c_2 .

Non-blocking: From every state where n_1 is true, there is a transition to a state where t_1 is true. Similarly for n_2 and t_2 .

No strict sequencing: From every state where c_1 is true, there is a path to a state where c_1 is false and then c_1 is true again without c_2 being true, and similarly with 1 and 2 reversed.

3.4:

1. (g)



5. Yes, because $\neg\text{busy}$ is true initially.
6. (b) (iv) $\mathcal{M}, s_0 \not\models \phi$ because t and q are both false in s_0 .
 $\mathcal{M}, s_2 \models \phi$ because q is true in s_2 .
6. (b) (viii) $\mathcal{M}, s_0 \models \phi$ and $\mathcal{M}, s_2 \models \phi$ because r or q is true in all states.
8. (b) $\mathcal{M}, s_0 \models \phi$ and $\mathcal{M}, s_2 \models \phi$ because from every state there is a path that leads to a state where p or r is true.