CS458/558 EXAM III April 9, 2015

Solutions

Credit will be given for partially correct answers, so it is to your advantage to explain your answers *clearly* and *concisely*. You may answer the questions in any order you choose. Just remember to number each problem clearly and write your name on the top of each sheet you turn in.

1. (30 points)

   (a) If $P(x, y)$, then $y = x, x + 1$, or $x + 2$. If $y = x$ or $x + 1$, taking $z = x$ makes ϕ true. If $y = x + 2$, taking $z = x + 1$ makes ϕ true.

   (b) Take $x = 2$, $y = 3$. Then there is no perfect square z^2 such that $Q(2, z^2) \wedge Q(z^2, 3)$.

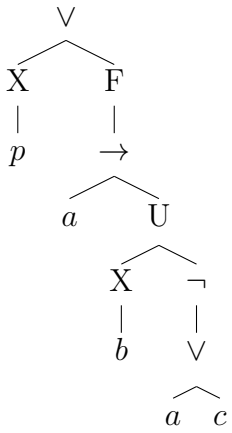
   (c) Take the model with universe $\{0, 1, 2\}$ and $P = \{(0, 1), (1, 2), (2, 0)\}$.

2. (20 points) Since $\mathrm{G}(\neg\phi \rightarrow \mathrm{X}(\neg\phi))$, once ϕ is false, it stays false.

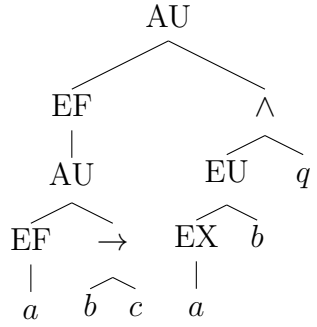
   Since $\mathrm{F}(\phi \wedge \mathrm{X}\psi)$, there is some step i where ϕ is true and ψ is true at the next step. So ϕ must be true for all steps $0, 1, \ldots, i$ and ψ is true at step $i + 1$. By definition, $\phi \mathrm{U} \psi$ is true.

3. (20 points)

   (a)



   (b)



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4. (30 points)

   (a) If $c \wedge d$, then the state must be s_2. The only possible next state is s_1, where b is true, so the formula is true.

   (b) The only state where b is true is s_1, and since the next states are s_3, s_2, $a \mathrm{U} c$ is not true, so the formula is false.

   (c) We need to show it's true for all paths starting with s_2. Since the future includes the inital state, the formula is true.

   (d) After s_0, there is no state where $a \wedge \neg b$, so the formula is false.

   (e) Any path starting with $s_1, s_3, s_2, \ldots$ satisfies the formula.

   (f) Not all paths satisfy the second clause EF AG c, e.g., any path starting with $s_3, s_1, s_1, s_1, \ldots$.