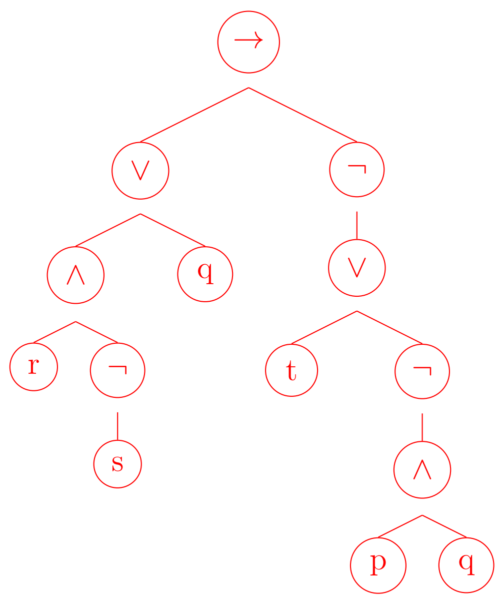
CS458/558 EXAM I February 5, 2015
Solutions

Credit will be given for partially correct answers, so it is to your advantage to explain your answers *clearly* and *concisely*. You may answer the questions in any order you choose. Just remember to number each problem clearly and write your name on the top of each sheet you turn in.

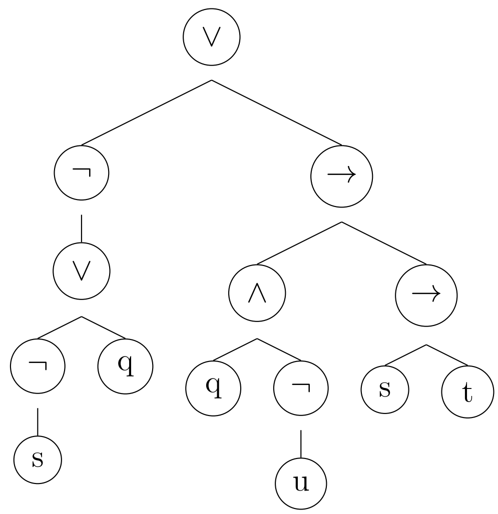
1. (20 points)

   (a) Draw the parse tree of the wff

$$(r \wedge \neg s) \vee q \rightarrow \neg(t \vee \neg(p \wedge q))$$



(b) Write the wff whose parse tree is shown below. You don't need to write the fully parenthesized form, but it should follow the rules for omitting parentheses in propositional logic.



$\neg(\neg s \vee q) \vee (q \wedge \neg u \to s \to t)$

2. (30 points) Show that

$$p \to q,\ q \to r \vdash \neg q \vee p \to \neg p \vee r$$

is a valid sequent. Do this in two ways:

(a) Use a truth table.

p	q	r	$p \to q$	$q \to r$	$\neg q \vee p \to \neg p \vee r$	
F	F	F	T	T	T	*
F	F	T	T	T	T	*
F	T	F	T	F	T	
F	T	T	T	T	T	*
T	F	F	F	T	F	
T	F	T	F	T	T	
T	T	F	T	F	F	
T	T	T	T	T	T	*

The sequent is valid because in every row of the truth table where all the premises are true, the conclusion is true. These rows are marked with a *. Note that it is not necessary to fill out the table completely. If you see that one of the premises in a row is F, then you don't need to fill out anything else in that row.

(b) Write a formal proof, using the rules of natural deduction summarized on p. 27 of the text.

1.	$p \to q$	premise
2.	$q \to r$	premise
3.	$\neg q \vee p$	assumption
4.	$\neg q$	assumption
5.	$\neg p$	MT 1,4
6.	$\neg p \vee r$	$\vee$i 5
7.	p	assumption
8.	q	MP 1,7
9.	r	$\to$ e 2,8
10.	$\neg p \vee r$	$\vee$i 9
11.	$\neg p \vee r$	$\vee$e 3, 4–6, 7–10
12.	$\neg q \vee p \to \neg p \vee r$	$\to$ i 3–11

3. (25 points)

(a) Show that

$$\neg p \to q \vdash q \to \neg p$$

is not a valid sequent. Which principle (soundness or completeness) are you using to show this?

The truth assignment $p =$ T, $q =$ T makes the premise T but the conclusion F. Therefore

$$\neg p \to q \not\models q \to \neg p$$

i.e., the premise does not semantically entail the conclusion. Therefore by the soundness principle,

$$\neg p \to q \nvdash q \to \neg p$$

(b) Show that

$$\models (r \wedge s) \vee t \to r \wedge (s \vee t)$$

does not hold.

The truth assignment $r =$ F, $s =$ T, $t =$ T makes the wff F, i.e., not a tautology.

4. (25 points)

(a) Suppose the rule

$$\frac{\neg(\phi \vee \psi)}{\neg\phi \wedge \neg\psi}$$

is added to the natural deduction rules on p. 27 of the text. Is the new system of deduction still sound? Explain.

Yes: whenever the premise of the rule is T, so is the conclusion. So in any proof where this rule is used, soundness will be preserved. You can also argue that the induction proof of soundness still holds when this rule is added as a new case.

(b) Suppose the rule

$$\frac{\phi \qquad \phi \vee \neg\psi}{\psi}$$

is added to the natural deduction rules on p. 27 of the text. Is the new system of deduction still sound? Explain.

No: let $\phi = p$ and $\psi = \neg p$. Then, using this rule,

$$p \vdash \neg p$$

(can you do it?) But

$$p \not\models \neg p$$

so adding this rule makes the system unsound. Lots of other examples could have been used.

(c) Suppose the rule $\vee$ e is removed from the collection of rules on p. 27. Is the system of deduction still sound? Explain.

Yes: the induction proof for soundness still works because there is one less case to deal with.

(d) Suppose the rule LEM is removed from the collection of rules on p. 27. Is the system of deduction still complete? Explain.

Yes: LEM is a derived rule, meaning it can be proved from the basic rules (see p. 25 of text). So any proof that uses it can be replaced by a proof that does not.