CS345/541 **Practice for Test 3** Fall 2025
MA345

Books, notes, calculators, computers and phones are *not* permitted.

1. (20%) Say that the memory of a Turing machine is *doubly infinite* if it is infinite in both directions. Show that a Turing machine with doubly infinite memory can be simulated by a basic Turing machine (with semi-infinite memory). [Exercise 7.4.2]

2. (20%) Let ALL_{DFA} be the language of strings of the form $\langle M \rangle$ where M is a DFA that accepts every possible string over its input alphabet. Show that ALL_{DFA} is decidable. (You can use, without proof, the other decidability results we covered in class.) [Exercise 8.2.1]

3. (20%) Consider the problem of detecting if a Turing machine M ever attempts to move left from the first position of its tape while running on w. This corresponds to the language $\text{BUMPS\_OFF\_LEFT}_{\text{TM}}$ of strings of the form $\langle M, w \rangle$ where M is a Turing machine, w is a string over the input alphabet of M and M attempts to move left from the first position of its tape while running on w. Show that $\text{BUMPS\_OFF\_LEFT}_{\text{TM}}$ is undecidable. (You can use, without proof, the other undecidability results we covered in class.) [Exercise 8.6.1]

4. (20%) Let $\text{INFINITE}_{\text{TM}}$ be the language of strings of the form $\langle M \rangle$ where M is a Turing machine and $L(M)$ is infinite. Show that $\text{INFINITE}_{\text{TM}}$ is undecidable. (You can use, without proof, the other undecidability results we covered in class. But you can't use Rice's Theorem.) [Exercise 8.6.7]

5. (20%) Let L be any language. Show that if both L and $\overline{L}$ are recognizable, then L and $\overline{L}$ are decidable. [Proof of Theorem 8.22]