

Given: Fri, Sep 15

Due: Fri, Sep 22, 9:00 a.m.

1. (32%) Give a formal description of the DFA of Figure 2.17, p. 23, in the notes. Describe the transition function by using equations.
2. (32%) This exercise asks you to show that DFA's can compare numbers when the numbers are presented in a particular way. For example, to compare 782 and 693, you would give to the DFA the input string that results from alternating the digits of the two numbers: 768923.

So design a DFA that when given two numbers  $x$  and  $y$  in this way determines if  $x \geq y$ . For example, the DFA should accept 768923 because  $782 \geq 693$ . But the DFA should reject 559913 because  $591 < 593$ .

More precisely, design a DFA that recognizes the following language: the set of strings of the form  $x_{n-1}y_{n-1} \cdots x_1y_1x_0y_0$  such that  $n \geq 1$  and if  $x$  is the number represented by  $x_{n-1} \cdots x_1x_0$  and  $y$  is the number represented by  $y_{n-1} \cdots y_1y_0$ , then  $x \geq y$ .

Note that the empty string and all strings of odd length should be rejected. The alphabet for this problem is the set of all digits, 0 through 9.

*Hint:* If you find it difficult to draw the DFA, don't forget that you can give a formal description and describe the transition function with equations. If useful, describe in words the role that each state plays in your DFA.

3. (36%) Consider the DFA's shown in Figures 2.12 and 2.16, pp. 20 and 22,

in the notes. Use the pair construction algorithm to obtain a DFA for the union of the languages of these two DFA's. Draw the resulting DFA.