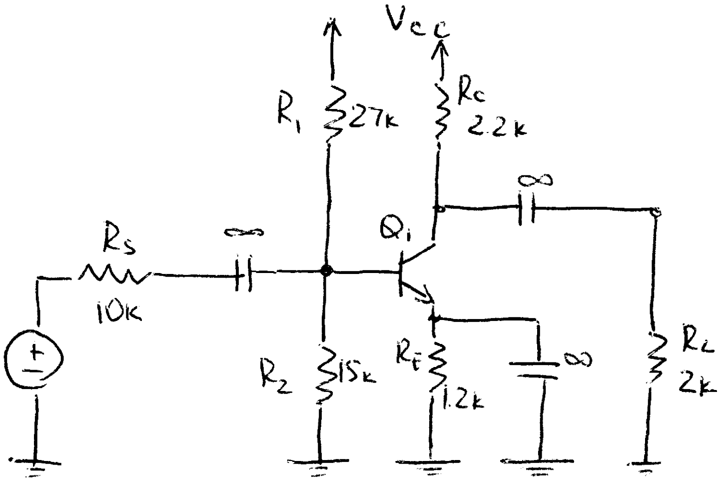
Assignment # 7

Problem 4.78



$\beta = 100$

$V_A = 100\ V$

$V_{CC} = 9\ V$

First, calculate dc bias emitter current I_E

Writing KVL for base-emitter: $V_{BB} - 0 = 0.7V + I_E R_E + I_B R_{BB}$

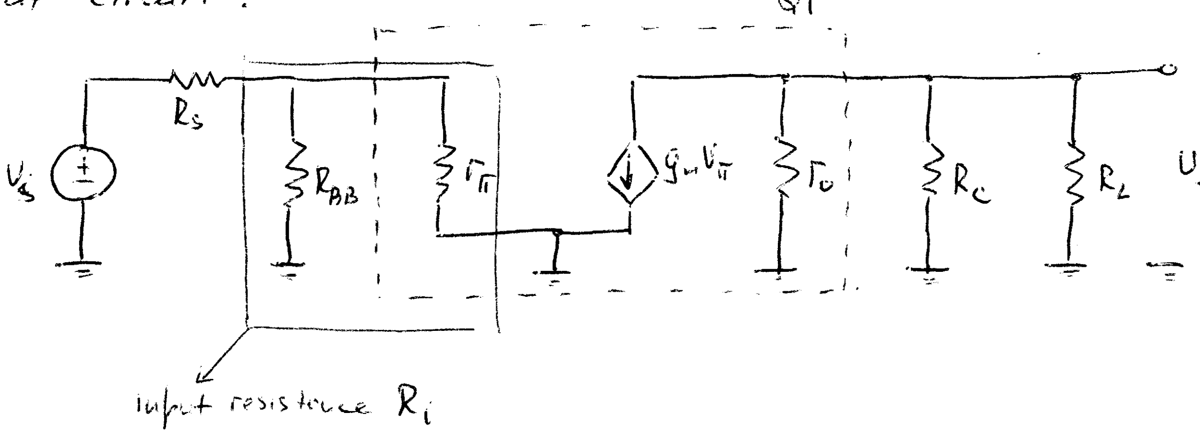
where V_{BB} and R_{BB} are found from: $V_{BB} = V_{CC} \frac{R_2}{R_1 + R_2} = 9V \frac{15k}{27k + 15k} \approx 3.21\ V$

$$R_{BB} = \frac{R_1 R_2}{R_1 + R_2} = \frac{15k \cdot 27k}{15k + 27k} = 9.64\,k\Omega$$

$$I_E = \frac{V_{BB} - 0.7V}{R_E + \frac{R_{BB}}{\beta+1}} = \frac{V_{CC}\frac{R_2}{R_1+R_2} - 0.7V}{R_E + \frac{R_1 R_2}{(\beta+1)(R_1+R_2)}} = \frac{3.21V - 0.7V}{1.2k\Omega + \frac{9.64k\Omega}{101}} \approx 1.94\ mA$$

$I_C = \alpha I_E \approx 1.92\ mA$

Equivalent circuit:



Now, we are able to find small signal characteristics:

transconductance g_m; r_π; r_o:

$$g_m = \frac{I_C}{V_T} = \frac{1.92\,mA}{25\,mV} = .0768\ A/V = 76.8\ mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{76.8\ mA/V} = 1.3\,k\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100\,V}{1.92\,mA} = 52.08\,k\Omega$$

Input resistance $R_i = R_{BB} \,//\, r_\pi$ (see equivalent circuit)

$$R_i = \frac{R_{BB}\, r_\pi}{R_{BB} + r_\pi} = \frac{9.64\,k\Omega \cdot 1.3\,k\Omega}{9.64\,k\Omega + 1.3\,k\Omega} = \underline{\underline{1.15\,k\Omega}}$$

The voltage gain $A_v = \frac{v_o}{v_s} = \frac{v_o}{v_\pi} \cdot \frac{v_\pi}{v_s}$

$$\frac{v_\pi}{v_s} = \frac{R_i}{R_i + R_s} \quad \text{(voltage divider)} \qquad \frac{v_o}{v_\pi} = -g_m \cdot (r_o \,//\, R_c \,//\, R_L)$$

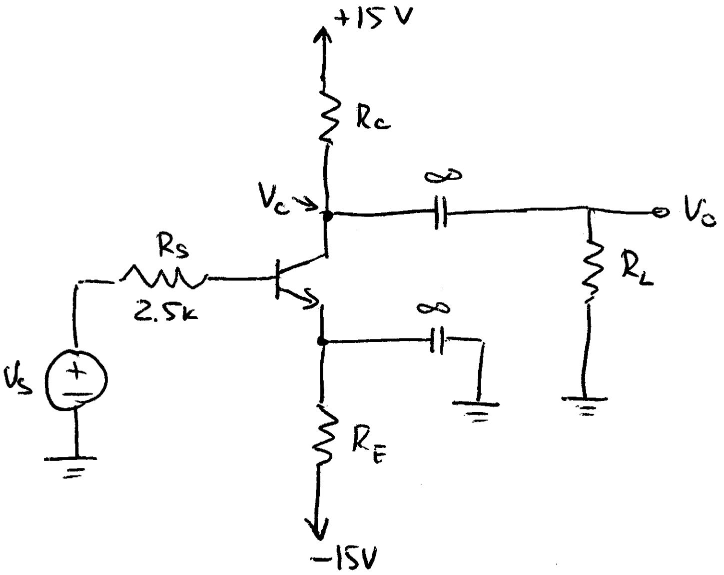
$$A_v = -\frac{g_m\left(R_i \cdot (r_o \,//\, R_c \,//\, R_L)\right)}{R_i + R_s} \qquad r_o \,//\, R_c \,//\, R_L = 1.027\,k\Omega$$

$$A_v = -\frac{76.8\ \frac{mA}{V} \cdot 1.15\,k\Omega \cdot 1.027\,k\Omega}{1.15\,k\Omega + 10\,k\Omega} = \underline{\underline{-8.135\ V/V}}$$

Current gain $A_i = \frac{i_o}{i_s} = \frac{v_o/R_L}{v_s/(R_s + R_i)} = \frac{v_o}{v_s} \cdot \frac{R_s + R_i}{R_L} = A_v \cdot \frac{R_s + R_i}{R_L} =$

$$= -8.135\ V/V \cdot \frac{10\,k\Omega + 1.15\,k\Omega}{2\,k\Omega} = \underline{\underline{-45.35\ A/A}}$$

Problem 4.82



$\beta = 100$

(a) Find R_E to establish dc $I_E = 1mA$

Writing DC KVL base-emitter:

$$0 - (-15V) = I_E R_E + 0.7V + I_B \cdot R_s$$

$$R_E = \frac{15V - 0.7V}{I_E} - \frac{R_s}{\beta+1} =$$

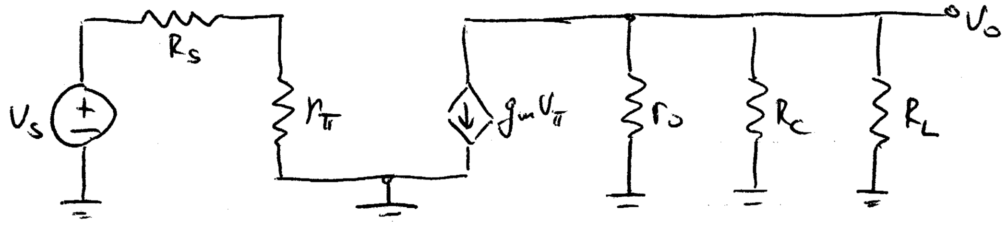
$$= \frac{14.3V}{1mA} - \frac{2.5k\Omega}{101} = \underline{14.28\,k\Omega}$$

(b) Find R_c to establish dc collector voltage +5 V

$$V_c = 15V - I_c R_c = 15V - \alpha I_E R_c \quad ; \quad R_c = \frac{15V - V_c}{\alpha I_E} = \frac{15V - 5V}{.99 \cdot 1mA} = \underline{10.1k\Omega}$$

(c) For $R_L = 5k\Omega$ and $r_o = 100k\Omega$ draw equivalent circuit and find A_v

Equivalent circuit:



Small signal characteristics: $g_m = \frac{I_c}{V_T} = \frac{.99mA}{25mV} = 39.6\ mA/V$

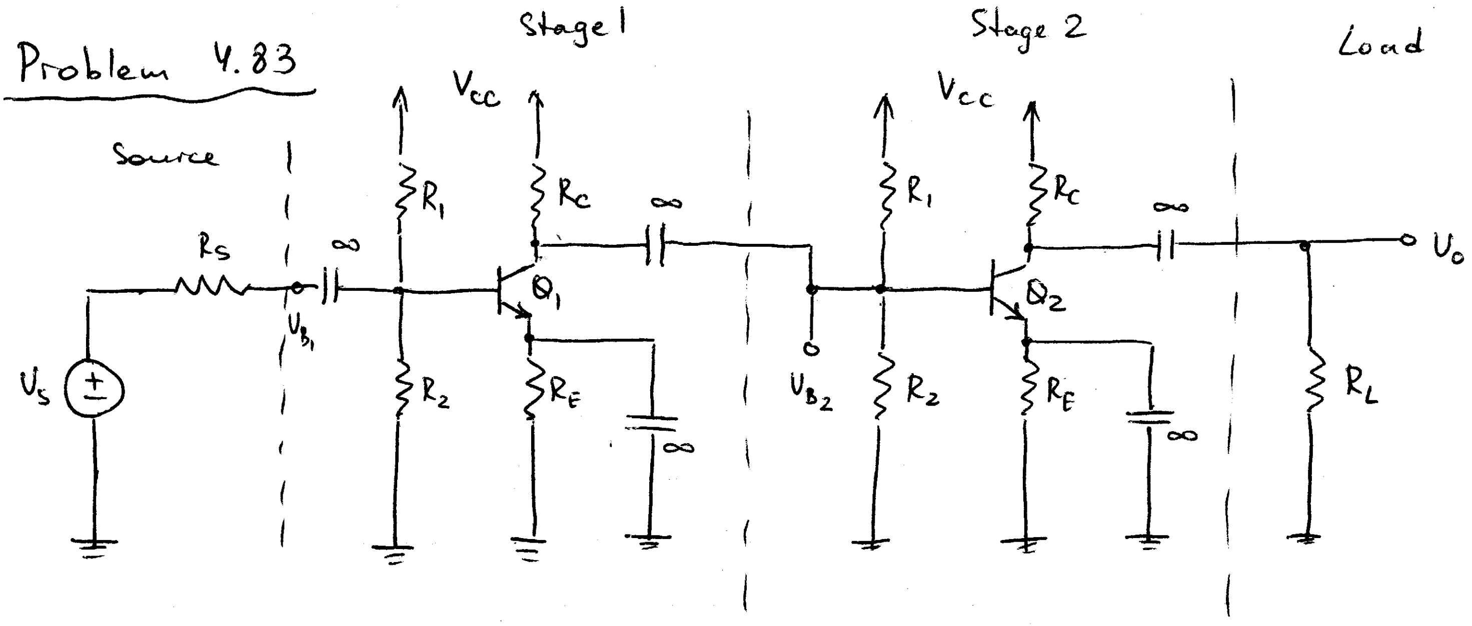
$r_\pi = \frac{\beta}{g_m} = \frac{100}{39.6\,mA/V} = 2.53\,k\Omega$ $\qquad r_o = 100k\Omega$ (given)

Voltage gain $A_v = \frac{V_o}{V_s} = \frac{V_o}{V_\pi}\frac{V_\pi}{V_s} = -g_m \cdot (r_o \| R_c \| R_L) \cdot \frac{r_\pi}{R_s + r_\pi}$

$r_o \| R_c \| R_L = 100k\Omega \,\|\, 10.1k\Omega \,\|\, 5k\Omega = 3.24k\Omega$ $\qquad \frac{V_o}{V_\pi}$ $\qquad \frac{V_\pi}{V_s}$ from voltage divider

$$A_v = -39.6\ mA/V \cdot (3.24k\Omega) \cdot \frac{2.53k\Omega}{2.5k\Omega + 2.53k\Omega} = \underline{-64.53\ V/V}$$

Problem 4.83



$V_{CC} = 15V \quad R_1 = 100k\Omega \quad R_2 = 47k\Omega \quad R_E = 3.9k\Omega \quad R_C = 6.8k\Omega \quad \beta = 100$

$R_S = 5k\Omega \quad R_L = 2k\Omega$

(a) Determine DC collector current and collector voltage of each transistor.

DC for Q_1, Q_2: $V_{BB} - 0V = I_B R_{BB} + 0.7V + I_E R_E$

V_{BB} and R_{BB} are found from: $V_{BB} = V_{CC} \cdot \frac{R_2}{R_1 + R_2} = 15V \cdot \frac{47k\Omega}{147k\Omega} = 4.8V$

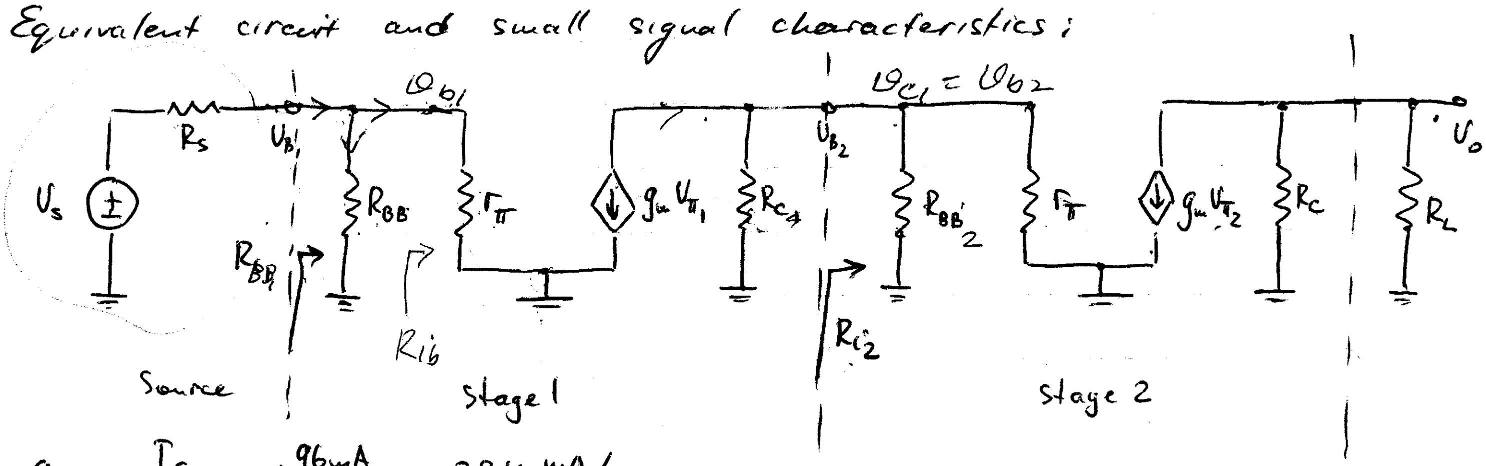
$$R_{BB} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{100k\Omega \cdot 47k\Omega}{147k\Omega} \approx 32k\Omega$$

$$I_E = \frac{V_{BB} - 0.7V}{R_E + \frac{R_{BB}}{\beta+1}} = \frac{4.8V - 0.7V}{3.9k\Omega + \frac{32k\Omega}{101}} \approx 0.97\ mA$$

$I_C = \alpha I_E = .99 \cdot 0.97\ mA = \underline{.96mA}$

$V_C = V_{CC} - I_C \cdot R_C = 15V - .96mA \cdot 6.8k\Omega = \underline{8.47\ V}$

(b) Equivalent circuit and small signal characteristics:



$$g_m = \frac{I_C}{V_T} = \frac{.96mA}{25mV} = 38.4\ mA/V$$

$R_{i_2} = R_{BB} // r_\pi = 2.4k\Omega$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{38.4\ mA/V} = 2.6k\Omega$$

$$R_{i_1} = R_{BB} // r_\pi = \frac{32k\Omega \cdot 2.6k\Omega}{32k\Omega + 2.6k\Omega} = \underline{\underline{2.4k\Omega}}$$

$$\frac{\text{Input Voltage Stage 1}}{\text{Signal voltage}} = \frac{v_{B1}}{v_s} = \frac{R_{i_1}}{R_s + R_{i_1}} = \frac{2.4\,k\Omega}{5k\Omega + 2.4k\Omega} = 0.324\ V/V$$

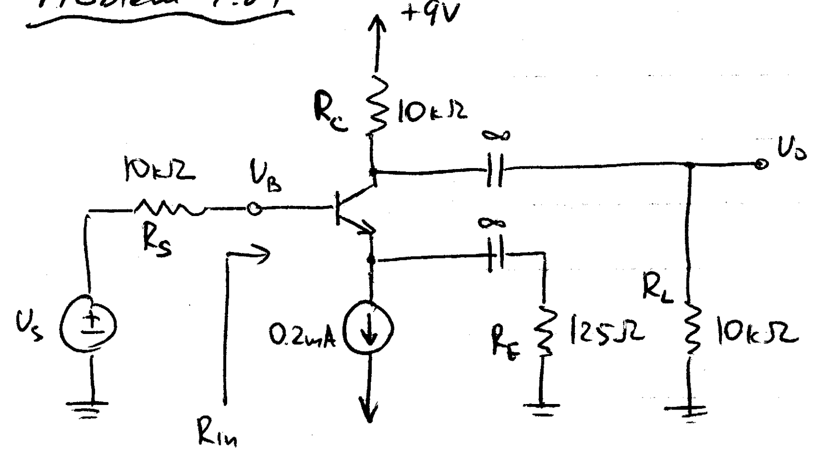
$$\frac{\text{Output Stage 1}}{\text{Input Stage 1}} = \frac{v_{B2}}{v_{B1}} = -g_m \cdot (R_C \parallel R_{BB} \parallel r_\pi) = -g_m (R_C \parallel R_i) =$$

$$= -38.4\ mA/V \cdot \frac{6.8k\Omega \cdot 2.4k\Omega}{6.8k\Omega + 2.4k\Omega} = -68.12\ V/V$$

$$\frac{\text{Output Load}}{\text{Input stage 2}} = \frac{v_o}{v_{B2}} = -g_m (R_C \parallel R_L) = -38.4\ mA/V \cdot \frac{6.8k\Omega \cdot 2k\Omega}{6.8k\Omega + 2k\Omega} = -59.35\ V/V$$

$$\frac{\text{Output load}}{\text{Input signal}} = \frac{v_o}{v_s} = \frac{v_o}{v_{B2}} \cdot \frac{v_{B2}}{v_{B1}} \cdot \frac{v_{B1}}{v_s} = -59.35\ V/V \cdot (-68.12\ V/V) \cdot 0.324\ V/V \doteq \underline{1309\ V/V}$$

Problem 4.84



v_s – small sine-wave signal with 0 average.

$\beta = 50$

$R_{in} = (\beta+1)(r_e + R_E)$

$r_e = \frac{V_T}{I_E} = \frac{25mV}{0.2mA} = 125\Omega$

given by current source

$$R_{in} = (50+1) \cdot (125\Omega + 125\Omega) = \underline{12.75\,k\Omega}$$

$$\frac{v_B}{v_s} = \frac{R_{in}}{R_{in}+R_s} = \frac{12.75k\Omega}{12.75k\Omega + 10k\Omega} = \underline{0.56\ V/V}$$

$$\frac{\text{Output}}{\text{Input}} = \frac{v_o}{v_B} = \frac{-\alpha \cdot (R_C \parallel R_L)}{r_e + R_E} = -\frac{0.98 \cdot 5k\Omega}{250\Omega} = \underline{-19.6\ V/V}$$

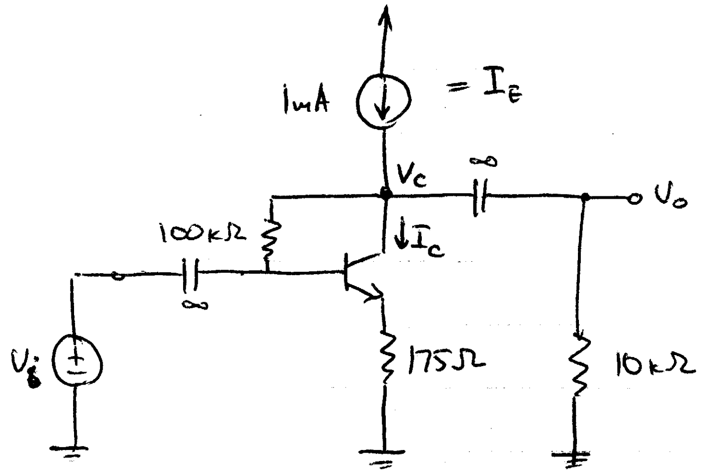
$$\frac{v_o}{v_s} = \frac{v_o}{v_B} \cdot \frac{v_B}{v_s} = -19.6\ V/V \cdot 0.56\ V/V = \underline{-10.98\ V/V}$$

If $v_{BE} = 5mV$ (it drops on r_e), then $v_{RE} = 5mV$ (since $R_E = r_e = 125\Omega$)

$v_B = v_{BE} + v_E = 10mV$. $\quad v_s = \frac{v_B}{(v_B/v_s)} = \frac{10mV}{.56V/V} = \underline{17.9\ mV}$

$v_o =$ ~~v_B~~ $v_s \cdot \left(\frac{v_o}{v_s}\right) = 17.9mV \cdot (-10.98\ V/V) = \underline{-196.5\ mV}$

Problem 4.85



$\beta = 100$

a) dc collector current

$$I_c = \frac{\beta}{\beta+1} \cdot 1mA = .99mA$$

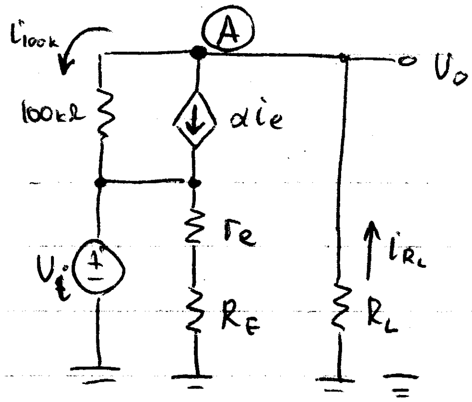
dc collector voltage

$$V_c = I_B \cdot 100k\Omega + 0.7V + I_E \cdot 175\Omega =$$

$$= 0.7V + I_E\left(175\Omega + \frac{100k\Omega}{101}\right) =$$

$$= \underline{\underline{1.87\ V}}$$

b) T-model:



From the figure,

$$V_i = i_e \cdot (R_E + r_e)$$

$$r_e = \frac{V_T}{I_E} = \frac{25mV}{1mA} = 25\Omega$$

Writing node equation for (A) $\quad i_{R_L} + \alpha i_e + i_{100k} = 0$

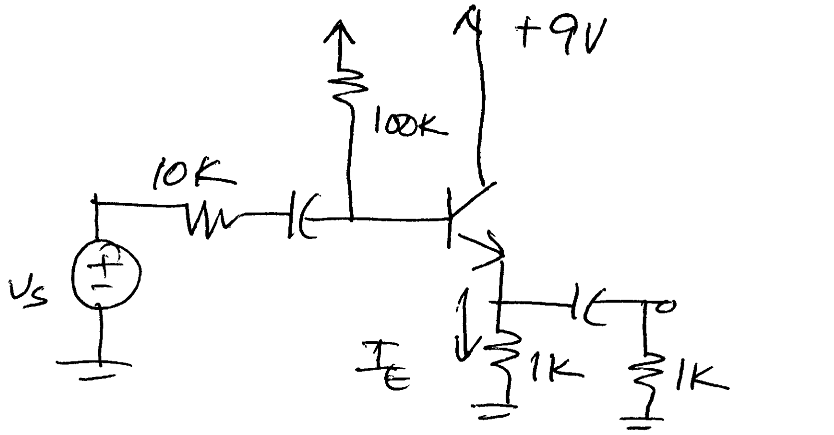
or $\frac{V_o}{R_L} + \alpha i_e + \frac{V_o - V_i}{100k\Omega} = 0$; $\frac{V_o}{R_L} + \alpha \cdot \frac{V_i}{R_E + r_e} + \frac{V_o - V_i}{100k\Omega} = 0$

$$V_o\left(\frac{1}{R_L} + \frac{1}{100k\Omega}\right) = V_i\left(\frac{1}{100k\Omega} - \frac{\alpha}{R_E + r_e}\right)$$

$$\frac{V_o}{V_i} = \frac{\left(\frac{1}{100k\Omega} - \frac{\alpha}{R_E + r_e}\right)}{\left(\frac{1}{R_L} + \frac{1}{100k\Omega}\right)} = \frac{\left(\frac{1}{100k\Omega} - \frac{0.99}{175\Omega + 25\Omega}\right)}{\left(\frac{1}{10k\Omega} + \frac{1}{100k\Omega}\right)} = -\frac{4.94}{0.11} =$$

$$= \underline{\underline{-44.9\ V/V}}$$

4.91 $I_E = \dfrac{9-0.7}{1K + \dfrac{100k}{\beta+1}}$



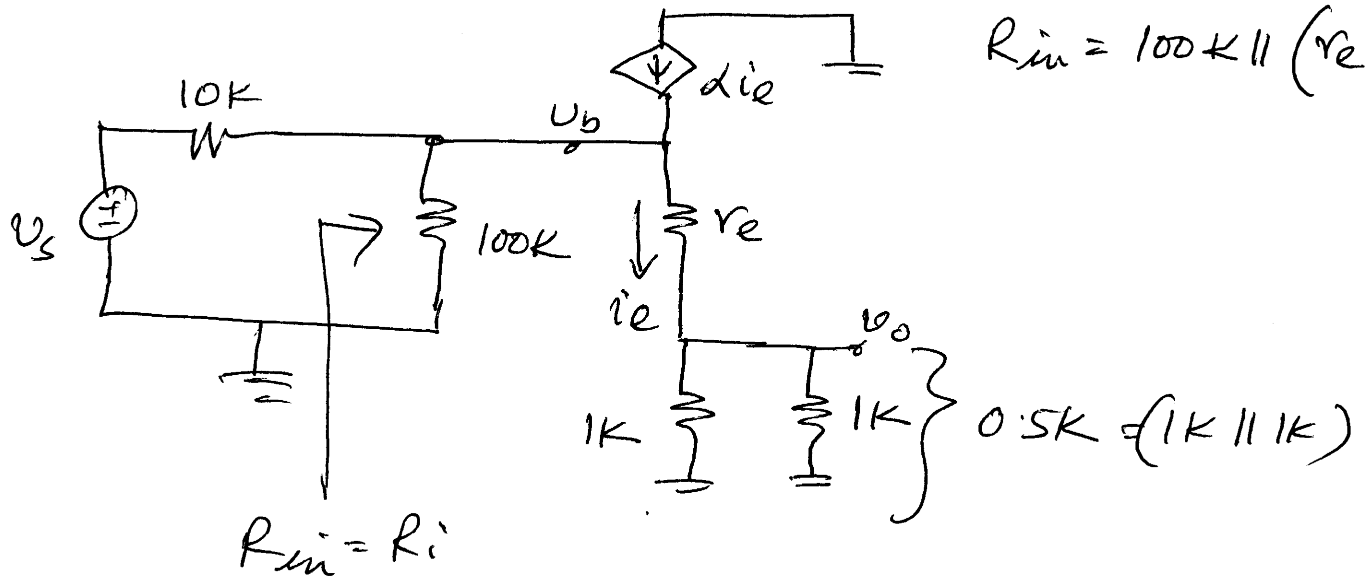
if $\beta = 20$:

$I_E = \dfrac{8.3}{1 + \dfrac{100}{21}} = 1.44\ mA$ $V_E = 1.44m \times 1K = 1.44V$

$V_B = V_E + 0.7 = 2.14V$ $< 9V$ Active

if $\beta = 200$ $I_E = 5.54\ mA, V_E = 5.54, V_B = 6.24 < 9V$ Active

Small signal equivalent ckt



$R_{in} = 100K \,||\, (r_e + 0.5K) * (1+\beta)$

if $\beta = 20$ $r_e = 17.4\ \Omega$ & $R_{in} = 9.8K\Omega$

if $\beta = 200$ $r_e = 4.51\ \Omega$ $R_{in} = 50.3K\Omega$

$\dfrac{v_o}{v_s} = \dfrac{v_o}{v_b} \cdot \dfrac{v_b}{v_o} = \dfrac{(1K||1K)}{r_e + (1K||1K)} \cdot \dfrac{R_{in}}{R_{in} + 10K}$

$\beta = 20;\ \dfrac{v_o}{v_s} = 0.478 \dfrac{V}{V}$

$\beta = 200;\ \dfrac{v_o}{v_s} = 0.827 \dfrac{V}{V}$