CLARKSON UNIVERSITY

Non-linear Aeroelasticity and Control of 2-D Lifting Surfaces

A Thesis by Christina M. Rubillo

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Submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

Accepted by the Graduate School

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Abstract

Linear and non linear aeroelastic modeling and control of 2-D lifting surfaces is the topic of this paper. Both two degree-of-freedom subsonic incompressible and 3 degree-of-freedom supersonic aeroelastic models with unsteady aerodynamics are explored. The effect of structural and aerodynamic nonlinearities on the aeroelastic response of the system is also outlined. Various solution methods, including the substitution of variables, perturbation method, harmonic balance, slowly varying amplitude and phase, and numerical integration methods are applied to a non-linear system. The harmonic balance method and numerical integration will be used to find the flutter speed and the aeroelastic response of the system. These calculated flutter boundaries are used to determine the stability of the system, as well as how structural parameters will change this boundary. Comparisons were made to previously presented works and the results were in good agreement.

Two aeroelastic control methodologies are presented in this paper: Dynamic Limiter Control (DLC) and Active Feedback Control (AFC). These studies will provide a better understanding of the non-linear dynamics of the open/closed-loop aeroelasticity of flexible wings with either steady or unsteady aerodynamic loads. The limiter control has been applied to control the plunging or pitching characteristic of the wing independently and to both of them simultaneously. On a prototypical wing section the control can effectively suppress Limit Cycle Oscillations (LCO) and chaos well beyond the nominal flutter speed. This could lead to a practical implementation of the control mechanism on actual and future generation aircraft wings via implementation of a combination of propulsive/jet type forces, micro surface effectors, and fluidic devices. Analysis of this
control produced favorable results in the suppression of LCO amplitude and increased flutter boundaries for plunging and pitching motion. The dynamic limiting control has asymptotically zero power and is simply implemented making it a feasible solution to the problem of the chaotic dynamics of the oscillating airfoil.

A linear and non-linear combined active proportional and velocity full state feedback control strategy is implemented. The control’s effectiveness in reducing the oscillation amplitude in the subcritical flight speed range, in suppressing flutter, and in preventing the catastrophic failure in the post-flutter range is demonstrated. Simulations were carried out using a time marching integration with the non-linear Mathematica® solver.

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