Power

- Power is energy per unit time
  \[ \text{POWER} = P = \frac{\text{FORCE} \times \text{DISTANCE}}{\text{TIME}} = FV_\infty \quad \text{if} \quad V_\infty \text{ is constant} \]

- For an airplane in level, unaccelerated flight, power \( P_R = T_R V_\infty \) required is
  \[
P_R = T_R V_\infty = \frac{W}{C_L/C_D} V_\infty \quad \text{but} \quad V_\infty = \sqrt{\frac{2W}{\rho S C_L}}
  \]

  - So
    \[
P_R = \frac{W}{C_L/C_D} \sqrt{\frac{2W}{\rho S C_L}} = \left( \frac{2W^3}{\rho S^2} \right)^{\frac{1}{2}} \left( \frac{C_D}{C_L} \right)\frac{\sqrt{C_L}}{\sqrt{C_D}}
    \]

  - That is,
    \[
P_R \propto \frac{1}{C_L^{3/2}/C_D}
    \]
Power

- How power required varies with drag

![Diagram showing power required vs. velocity with minimum power required curve highlighted.]

**AIRPLANE DATA**
- \( W = 15,000 \text{ lbs} \)
- \( b = 40 \text{ ft} \)
- \( e = 0.827 \text{ Sea Level} \)

\( f = 10.8 \text{ sq. ft.} \)  
\( f = 7.2 \text{ sq. ft.} \)

**Minimum Power Required**

**Power**

- Let’s analyze minimum power required
  - As we observed before for a parabolic drag polar
    \[
    P_k = T_k V_w = D V_w = q_w S \left( C_{D0} + \frac{C_l^2}{\pi e A R} \right) V_w
    \]
    \[
    P_k = q_w S C_{D0} V_w + q_w S V_w \frac{C_l^2}{\pi e A R}
    \]
    Parasite power required
    Induced power required
  - In level, unaccelerated flight
    \[
    C_L = \frac{2L}{\rho V^2 S} = \frac{2W}{\rho V^2 S};
    \]
    \[
    P_k = \frac{1}{2} \rho V^3 S C_{D0} + \frac{1}{2} \rho V^3 S \frac{C_l^2}{\pi e A R} = \frac{1}{2} \rho V^3 S C_{D0} + \frac{1}{2} \rho V^3 S + \frac{1}{2} \rho V^3 S \frac{4W^2}{\rho V^2 S \pi e A R}
    \]
    \[
    P_k = \frac{1}{2} \rho V^3 S C_{D0} + \frac{2W^2}{\rho V S \pi e A R}
    \]
**Power**

- Still looking for the minimum $P_R$ point
  - Differentiating this last expression for $P_R$ and setting this result to zero
    \[
    \frac{dP_R}{dV_w} = \frac{3}{2} \rho \frac{V_w^2 S C_{D_0}}{\rho V_w^2 S \pi eAR} - \frac{2W^2}{\rho V_w^2 S \pi eAR} \\
    = \frac{3}{2} \rho \frac{V_w^2 S}{C_{D_0}} - \frac{4W^2}{3 \rho \frac{V_w^2 S}{\pi eAR}} \\
    = \frac{3}{2} \rho \frac{V_w^2 S}{C_{D_0}} - \frac{C_i}{3 \pi eAR} \\
    = \frac{3}{2} \rho \frac{V_w^2 S}{C_{D_0} - \frac{C_i}{3}}
    \]
  - Thus, the relationship between $C_{D_0}$ and $C_{D_i}$ at minimum power required is
    \[
    C_{D_0} = \frac{C_{D_i}}{3}
    \]

- In level flight, $D = T_R$
Altitude effects on $P_R$

- Power required and velocity for level unaccelerated flight
  - At sea level
    \[ V_0 = \sqrt{\frac{2W}{\rho_s SC_L}} \Rightarrow P_{R,0} = \sqrt{\frac{2W^2 C_L^2}{\rho_s SC_L}} \]
  - At altitude
    \[ V_{alt} = \sqrt{\frac{2W}{\rho SC_L}} \Rightarrow P_{R,alt} = \sqrt{\frac{2W^2 C_L^2}{\rho SC_L}} \]
  - Dividing each altitude equation by the s.l. one
    \[ V_{alt} = V_0 \sqrt{\frac{\rho_s}{\rho}} = V_0 \sqrt{\frac{T}{\sigma}} \Rightarrow \frac{P_{R,alt}}{P_{R,0}} = \sqrt{\frac{T}{\sigma}} \]
  - On a power required curve, any point associated with a given $C_L$ at sea level, moves to the right and up as altitude is increased
Weight effects on $P_R$

\[
P_R = \frac{W}{C_L/C_D \sqrt{\frac{2W}{\rho S C_L}}}
\]

\[
= W^{\frac{1}{2}} \sqrt{\frac{2}{\rho S \left(\frac{1}{C_L/C_D}\right)}}
\]

\[
V = \sqrt{\frac{2W}{\rho S C_L}}
\]

- Define

\[
\omega \equiv \frac{W}{W_{oul}}
\]

\[
V = V_{oul} \sqrt{\omega}
\]

\[
P_R = P_{oul} \sqrt{\omega}
\]

Power available $P_A$

- Propeller efficiency is important for any powerplant driving a propeller \[ P_A = \eta P \]

- $\eta$ is propeller efficiency
- $P$ is brake horsepower
- $P_A$ is available horsepower $< P$

1 hp = 550 ft - lb / sec = 746 watts

\[ P_A = \eta P \]

Reciprocating engine-propeller combination
Power available $P_A$

- Power available from a jet engine has quite a different shape

$$ P_A = T_A V_w $$

Power available $P_A$

- Altitude effects on $P_A$ are kin to $P_R$
  - Propeller aircraft

Diagram of $P_A$ vs $V_w$ for jet and propeller aircraft.
Power available $P_A$

- Similarly for a jet-powered airplane