Steady Flight

Aerodynamic efficiency

- Lift/drag ratio is a measure of aerodynamic efficiency
  - It indicates the ability to produce lift without generating excessive drag

\[
\begin{align*}
  \text{MAX } L/D & \quad \text{VEHICLE} \\
  0.3-0.4 & \quad \text{GEMINI} \\
  12.8 & \quad \text{T-38} \\
  26 & \quad \text{SAILPLANE}
\end{align*}
\]

- Character of drag

\[
\begin{align*}
  C_L &= \frac{2L}{\rho V^2 S} = \frac{2W}{\rho V^2 S} \\
  C_D &= C_{D_b} + \frac{C_l^2}{\pi e AR} = C_{D_b} + KC_L^2 = C_{D_b} + \frac{4W^2}{\rho V^2 S^2 \pi e AR} \\
  D &= \frac{1}{2} \rho V^2 SC_{D_b} + \frac{2W^2}{\rho V^2 S \pi e AR} = \frac{1}{2} \rho V^2 SC_{D_b} + \frac{1}{2} \rho V^2 SKC_L^2
\end{align*}
\]

| Zero lift drag | Drag due to lift |
To maintain certain speed and altitude, enough thrust must be generated to overcome the drag and to keep the airplane going.
Drag = $T_R$

$$D = qSC_D = \frac{1}{2} \rho V^2 S \left[ C_{D_0} + KC_L^2 \right] = \frac{1}{2} \rho V^2 S \left[ C_{D_0} + \frac{4KW^2}{(\rho V^2 S)^2} \right]$$

Since for $L=W$

$$C_L = \frac{2W}{(\rho V^2 S)}$$

$$D = \frac{1}{2} \rho V^2 SC_{D_0} + 2SK \left( \frac{W}{S} \right) = f(h, V, W)$$

$$K = \frac{1}{e\pi AR}$$

Important parameters:
- Thrust to Weight ratio $T_R/W$
- Wing loading $W/S$
- Polar drag ($C_{D_0}$ and $K$)
Aerodynamic efficiency

- Relationship of $C_{D0}$ and $C_{D_i}$ for $L/D_{max}$

$$T_R = C_{D_b}q_wS + \frac{W^2}{q_wS\pi eAR}$$

- Differentiating with respect to $q_w$:

$$\frac{dT_R}{dq_w} = C_{D_b}S - \frac{W^2}{q_w^2S\pi eAR} = 0$$

- Solving for $C_{D_b}$

$$C_{D_b} = \frac{W^2}{q_w^2S\pi eAR} = \frac{KW^2}{q_w^2S^2}$$

- And observing that

$$\frac{W^2}{q_w^2S^2} = \left(\frac{W}{q_wS}\right)^2 = C_L^2$$

- Thus, at max $L/D$

$$C_{D_0} = \frac{C_L^2}{\pi eAR} = C_{D_i}$$
Equation (5.12) can be used to find the flight velocities for a given value of \( T_R \). Writing Eq. (5.12) in terms of the dynamic pressure \( q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 \) and noting that \( D = T_R \), we obtain

\[
T_R = q_\infty S C_{D,0} + \frac{K S}{q_\infty} \left( \frac{W}{S} \right)^2
\]

Multiplying Eq. (5.13) by \( q_\infty \) and rearranging, we have

\[
q_\infty^2 S C_{D,0} - q_\infty T_R + K S \left( \frac{W}{S} \right)^2 = 0
\]

Note that, being a quadratic equation in \( q_\infty \), Eq. (5.14) yields two roots, that is, two solutions for \( q_\infty \). Solving Eq. (5.14) for \( q_\infty \) by using the quadratic formula results in

\[
q_\infty = \frac{T_R \pm \sqrt{T_R^2 - 4SC_{D,0}K(W/S)^2}}{2SC_{D,0}}
\]

By replacing \( q_\infty \) with \( \frac{1}{2} \rho_\infty V_\infty^2 \), Eq. (5.15) becomes

\[
V_\infty^2 = \frac{T_R}{S} \pm \frac{\sqrt{(T_R/S)^2 - 4C_{D,0}K(W/S)^2}}{\rho_\infty C_{D,0}}
\]

The parameter \( T_R/S \) appears in Eq. (5.16); analogous to the wing loading \( W/S \), the quantity \( T_R/S \) is sometimes called the thrust loading. However, in the hierarchy of parameters important to airplane performance, \( T_R/S \) is not quite as fundamental as the wing loading \( W/S \) or the thrust-to-weight ratio \( T_R/W \) (as will be discussed in the next section). Indeed, \( T_R/S \) is simply a combination of \( T_R/W \) and \( W/S \) via

\[
\frac{T_R}{S} = \frac{T_R}{W} \frac{W}{S}
\]

Substituting Eq. (5.17) into (5.16) and taking the square root, we have our final expression for velocity:

\[
V_\infty = \left[ \frac{(T_R/W)(W/S) \pm (W/S)\sqrt{(T_R/W)^2 - 4C_{D,0}K}}{\rho_\infty C_{D,0}} \right]^{1/2}
\]
When the discriminant in Eq. (5.18) equals zero, then only one solution for $V_{\infty}$ is obtained. This corresponds to point 3 in Fig. 5.9, namely, the point of minimum $T_R$. That is, in Eq. (5.18) when

$$\left(\frac{T_R}{W}\right)^2 = 4C_{D,0}K = 0 \quad [5.19]$$

then the velocity obtained from Eq. (5.18) is

$$V_{\infty} = \left[\frac{1}{\rho_0 C_{D,0}} \frac{T_R}{W} \frac{W}{S}\right]^{1/2} \quad [5.20]$$

The value of $(T_R/W)_{\text{max}}$ is given by Eq. (5.19) as

$$\left(\frac{T_R}{W}\right)_{\text{max}} = 4C_{D,0}K$$

or

$$\left(\frac{T_R}{W}\right)_{\text{min}} = \sqrt{4C_{D,0}K} \quad [5.21]$$

Substituting Eq. (5.21) into Eq. (5.20), we have

$$V_{\infty} = \left(\frac{\sqrt{4C_{D,0}K}}{\rho_0 C_{D,0}} \frac{W}{S}\right)^{1/2}$$

or

$$V_{\infty} = \sqrt{\frac{2K}{\rho_0 C_{D,0} S}} \quad [5.22]$$

In Eq. (5.22), by stating that $V_{\infty} = V_{\infty,\text{min}}$, we are recalling that the velocity for minimum $T_R$ is also the velocity for maximum $L/D$, as shown in Fig. 5.6. Indeed, since $T_R = D$ and $L = W$ for steady, level flight, Eq. (5.21) can be written as

$$\left(\frac{D}{L}\right)_{\text{min}} = \sqrt{4C_{D,0}K} \quad [5.23]$$

Since the minimum value of $D/L$ is the reciprocal of the maximum value of $L/D$, then Eq. (5.23) becomes

$$\left(\frac{L}{D}\right)_{\text{max}} = \frac{1}{\sqrt{4C_{D,0}K}} \quad [5.24]$$
Effect of weight on $T_R$

- Drag
  \[ D = \frac{C_D_0 \rho V^2 S}{2} + \frac{2W^2}{\rho V^2 S \pi e AR} \]
  - Increasing aircraft weight by $\Delta w$
  \[ D = \frac{C_D_0 \rho V^2 S}{2} + \frac{2(W + \Delta W)^2}{\rho V^2 S \pi e AR} \]
  - Then
  \[ D = \frac{C_D_0 \rho V^2 S}{2} + \frac{2W^2}{\rho V^2 S \pi e AR} + \frac{2(2W \Delta W + \Delta W^2)}{\rho V^2 S \pi e AR} \]

Effect of altitude on $T_R$

- Multiplying the drag equation by $\frac{\rho}{\rho}$
  \[ D = \frac{C_D_0 \rho \rho V^2 S}{2} + \frac{2W^2}{\rho \rho V^2 S \pi e AR} \]
  - Does not change $T_{R \min}$
  - Minimum drag occurs at a higher $V_w$
  - $T_R$ curve opens up and shifts to the right
Thrust Available

- Thrust required is dictated by the airframe
  - Shape (airfoil, planform, fuselage, empennage)
  - Size (surface area, frontal area, airfoil)
  - Configuration (clean, gear down, flaps down)
- Thrust available is dictated by the powerplant (engine type, prop)
  - Reciprocating engine-propeller combination
  - Turbojet
  - Turboprop (turbine engine and propeller)
  - Turbofan
  - Ducted propeller
  - Rocket

Thrust Available

- Accelerating a mass of air gives $T_A$
  - Propeller operates on a large volume of air and imparts a small change in velocity
  - A turbojet operates on a smaller mass of air and imparts a larger change in velocity
Thrust Available

- $T_A$ and $T_R$ curve intersections give $V_{\text{max}}$
  - This statement is true for only a single altitude, since $T_A$ and $T_R$ are both altitude dependent
  - $T_A$ goes down as altitude increases

Importance of the Ratios $C_L/C_D, C_L^{3/2}/C_D, C_L^{1/2}/C_D$

- $\frac{C_L}{C_D}|_{\text{max}}^{\text{max}}$  Max Range for reciprocating engine/propeller airplanes
- $\frac{C_L^{3/2}}{C_D}|_{\text{max}}^{\text{max}}$  Max Endurance for jet-propelled airplanes
- $\frac{C_L^{1/2}}{C_D}|_{\text{max}}^{\text{max}}$  Max Endurance for reciprocating engine/propeller
- $\frac{C_L^1}{C_D}|_{\text{max}}^{\text{max}}$  Max Range for jet-propelled airplanes
Aerodynamic Relations Associated with Maximum $C_L/C_D$, $C_L^{3/2}/C_D$, and $C_L^{1/2}/C_D$

\[
\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_{L,0} + KC_L^2}{C_{D,0}}
\]

\[
d\left(\frac{C_L}{C_D}\right) = 0 \Rightarrow \quad C_{D,0} = KC_L^2
\]

\[
\left(\frac{L}{D}\right)_{\text{max}} = \left(\frac{C_L}{C_D}\right)_{\text{max}} = \sqrt{\frac{1}{4C_{D,0}K}}
\]

\[
L = W = \frac{1}{2} \rho_{\infty} V_{\infty}^2 SC_L
\]

\[
V_{(L/D)_{\text{max}}} = \left(\frac{2 \rho_{\infty} K}{C_{D,0} S}\right)^{1/2}
\]

Aerodynamic Relations Associated with Maximum $C_L/C_D$, $C_L^{3/2}/C_D$, and $C_L^{1/2}/C_D$

\[
(C_L^{1/2}/C_D)_{\text{max}} \Rightarrow \quad C_{D,0} = 3KC_L^2
\]

\[
\left(\frac{C_L^{1/2}}{C_D}\right)_{\text{max}} = \frac{3}{4} \left(\frac{1}{3KC_{D,0}}\right)^{1/4}
\]

\[
V_{C_L^{1/2}/C_D_{\text{max}}} = \frac{2 \rho_{\infty} K}{C_{D,0} S}^{1/2}
\]

\[
V_{(C_L^{1/2}/C_D)_{\text{max}}} = 3^{1/4} V_{(L/D)_{\text{max}}}
\]

\[
V_{C_L^{3/2}/C_D_{\text{max}}} < V_{C_L^{1/2}/C_D_{\text{max}}} < V_{C_L^{3/2}/C_D_{\text{max}}}
\]

Figure 5.11 Variation of $C_L^{3/2}/C_D$, $C_L/C_D$, and $C_L^{1/2}/C_D$ versus velocity for the Gulfstream IV in the conditions set in Example 5.1. Altitude = 30,000 ft, $W = 73,000$ lb.