Takeoff and Landing

Takeoff Performance

- \( V=0 \) (\( s=0 \))
- \( V=V_{\text{stall}} \)
- \( V=V_{\text{mcg}} \) (min control speed on the ground)
- \( V=V_{\text{mca}} \) (min control speed in the air (w/o landing gear in contact with ground))
- \( V=V_{\text{1}} \) decision speed (or critical engine failure speed)
  - balance speed length
- \( V=V_{\text{R}} \) (takeoff rotation speed) \( L \geq W \)
- \( V=V_{\text{MU}} \) (min unstick speed, ground clearance) \( \alpha \leq \alpha_{\text{cl tail}} \)
- \( V=V_{\text{LO}}=1.1 \ V_{\text{stall}} \) (\( s=s_g \)) (lift off speed)

\[ s = s_g + s_a \]

airborne but before it clears an obstacle of height \( h^* \)

\[ s = s_g \]
Takeoff Speed and FAR 25 requirements

<table>
<thead>
<tr>
<th>Speed</th>
<th>Description</th>
<th>FAR 25 Requirement</th>
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<tr>
<td>$V_s$</td>
<td>stall speed in takeoff configuration</td>
<td>-</td>
</tr>
<tr>
<td>$V_{mc}$</td>
<td>minimum control speed with one engine inoperative (OEI)</td>
<td>-</td>
</tr>
<tr>
<td>$V_1$</td>
<td>OEI decision speed</td>
<td>$= or &gt; V_{mc}$</td>
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<tr>
<td>$V_r$</td>
<td>rotation speed</td>
<td>$5% &gt; V_{mc}$</td>
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<tr>
<td>$V_{mu}$</td>
<td>minimum unstick speed for safe flight</td>
<td>$= or &gt; V_s$</td>
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<tr>
<td>$V_{lof}$</td>
<td>liftoff speed</td>
<td>$10% &gt; V_{mu}$ (OEI)</td>
</tr>
<tr>
<td>$V_2$</td>
<td>takeoff climb speed at 35 ft</td>
<td>$20% &gt; V_s$</td>
</tr>
</tbody>
</table>

Examples of Takeoff Speeds

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Takeoff Weight</th>
<th>Takeoff Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boeing 737</td>
<td>100,000 lb 45,360 kg</td>
<td>150 mph 250 km/h 130 kts</td>
</tr>
<tr>
<td>Boeing 757</td>
<td>240,000 lb 108,860 kg</td>
<td>160 mph 260 km/h 140 kts</td>
</tr>
<tr>
<td>Airbus A320</td>
<td>155,000 lb 70,305 kg</td>
<td>170 mph 275 km/h 150 kts</td>
</tr>
<tr>
<td>Airbus A340</td>
<td>571,000 lb 259,000 kg</td>
<td>180 mph 290 km/h 155 kts</td>
</tr>
<tr>
<td>Boeing 747</td>
<td>800,000 lb 362,870 kg</td>
<td>180 mph 290 km/h 155 kts</td>
</tr>
<tr>
<td>Concorde</td>
<td>400,000 lb 181,435 kg</td>
<td>225 mph 360 km/h 195 kts</td>
</tr>
</tbody>
</table>
The critical engine speed defines the point on the runway at which the distance needed to stop is exactly the same as the that required to reach takeoff speed. The resulting total takeoff distance is correspondingly known as the balanced field length.

**Definition of critical engine-failure speed and balanced field length**

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**EOM Airplane During Takeoff**

This equation gives instantaneous forces during the acceleration:

\[ F = T - D - R = T - D - \mu_r (W - L) = m \frac{dV}{dt} \]

\[ R = \mu_r (W - L) \]

- **\( P = V_r T \)**
- **\( T = \frac{\text{const}}{V_r} \)** reciprocating engine/propeller
- **\( T = \text{const} \)** turbojet
- **\( T = k_1 V_r + k_2 V_r^2 \)** turbofan
- **\( W = \text{const} \)** Weight

\( k_1, k_2 \) are constants.
**EOM Airplane During Takeoff**

- Both $L$ and $D$ vary with $V$

$$L = \frac{1}{2} \rho \infty V^2 SC_L \quad \quad D = \frac{1}{2} \rho \infty V^2 SC_D$$

$$C_D = C_{D0} + \Delta C_{D0} + \left( k_1 + Gk_2 \right) C_L^2$$  

**Drag Polar**  

$$k_2 = 0 \quad \text{Wave Drag}$$

$$\Delta C_{D0} = \frac{W}{S} K_{uc} m^{-0.219} \quad W/ S \quad \text{[N/m}^2] \quad m \quad \text{max mass aircraft [kg]}$$

$$K_{uc} = 5.81 \cdot 10^{-5}, \quad K_{uc} = 3.16 \cdot 10^{-5}$$

- **“Ground Effect”**

$$\frac{C_D \text{ (in ground effect)}}{C_D \text{ (out-of-ground effect)}} = G \quad G = \frac{(16h/b)^2}{1+(16h/b)^2}$$

$h$ = height of wing above the ground, $b$ = wingspan

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**EOM Airplane During Takeoff**

limited and determined by features of the geometric design configuration of the airplane rolling along the ground

$$L = \frac{1}{2} \rho \infty V^2 SC_L \quad C_L \leq 0.1$$

$$D = \frac{1}{2} \rho \infty V^2 SC_D$$

$$C_D = C_{D0} + \Delta C_{D0} + \left( k_1 + Gk_2 \right) C_L^2$$

**Drag Polar**  

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**Ground Roll Distance**

$$ds = \frac{ds}{dt} dt = V_\infty dt$$

$$\int_0^t ds = \int_0^t V_\infty dt \quad \Rightarrow \quad s_{eq} = \int_0^t V_\infty dt$$
Average Forces Acting During Takeoff

\[ F_{\text{eff}} = T - [D + \mu_r (W - L)]_{\text{average}} = \text{CONSTANT} \]

- Why consider “average” force during the takeoff roll?
  - average \( 0.7 \ V_L \)

Takeoff Roll
- We are no longer considering a “Statics” problem
  - Finite (even large) accelerations are present
  - we apply Newton’s second law to any body initially at rest,
    
    \[
    \begin{align*}
    F &= \ma = m \frac{dV}{dt} \\
    \int_0^V dV &= \int_0^t \frac{F_{\text{eff}}}{m} dt \Rightarrow V = \left( \frac{F_{\text{eff}}}{m} \right) t \\
    \int_0^t dt &= \frac{F_{\text{eff}} t}{m} \quad \text{OR} \quad t = \left( \frac{V m}{F_{\text{eff}}} \right) \\
    \int_0^V V dt &= \int_0^t \frac{F_{\text{eff}}}{m} t dt = \frac{F_{\text{eff}} t^2}{m} \int_0^t dt \Rightarrow s = \left( \frac{F_{\text{eff}}}{m} \right) \frac{t^2}{2}
    \end{align*}
    \]
Approximate Analysis Ground Roll

\[ ds = \frac{ds}{dt} = V_0 \frac{dt}{dV_0} = V_0 \frac{dV_0}{dV_a} \quad ds = V_a \left( \frac{dV_a}{dV_a / dt} \right) = \frac{1}{2} \left( \frac{d(V_a^2)}{dV_0} \right) \]

\[ \frac{dV_a}{dt} = \frac{1}{m} \left( T - D - \mu \left( W - L \right) \right) = \frac{g}{W} \left( T - D - \mu \left( W - L \right) \right) \]

\[ s_N = \int_0^{V_{lo}} \frac{d \left( V_a^2 \right)}{2g \left( K_T - K_A V_a^2 \right)} = \int_0^{V_{lo}} \frac{W}{2g \left( T - D - \mu \left( W - L \right) \right)} + NV_{LO} \]

\[ K_T = \frac{T}{W} - \mu \]

\[ K_A = \frac{\rho_a}{2(W/S)} \left( C_{D_0} + \Delta C_{D0} + \left( k + \frac{G}{\pi eAR} \right) C_L^2 - \mu C_L \right) \]

\[ N = 3 \quad \text{large aircraft;} \quad N = 1 \quad \text{small aircraft;} \]

\[ NV_{LO} \quad \text{distance covered during rotation} \]

Takeoff Roll

- Lift-off distance for a jet airplane
  - Using the average Force we have postulated
    - The sum
      \[ F_{\text{avg}} = T - \left[ D + \mu \left( W - L \right) \right] \]
    - Is fairly constant
    - Thrust is also fairly constant
  - Substituting into
    \[ D + \mu \left( W - L \right) \]

\[ s = \frac{V_0^2 m}{2F_{\text{avg}}} \quad s_{LO} = \frac{V_{LO}^2 W}{2g F_{\text{avg}}} \]

\[ s_{LO} = \frac{V_{LO}^2 W}{2g \left( T - \left[ D + \mu \left( W - L \right) \right] \right)} + NV_{LO} \]
Takeoff Roll

- Lift-off distance (continued)
  - Generally we use a safety margin at lift-off by 10%
    \[ V_{LO} = 1.1V_{null} = 1.1 \sqrt{\frac{2W}{\rho_a S C_{T_{max}}}} \]
  - Substituting into the expression for \( s_{LO} \)
    \[ s_{LO} = \frac{1.21(W/S)}{g \rho_a C_{T_{max}} \left\{ T/W - \left[ D/W + \mu_r (1 - L/W) \right]_{\text{average}} \right\}} + 1.1N \left( \frac{2}{\rho_a C_{T_{max}}} \right) \left( \frac{W}{S} \right) \]
  - One method of estimating quickly the average drag and rolling resistance force is to use \( 0.7V_{LO} \) in calculating the aerodynamic forces

Takeoff Ground Roll Distance – Simplified Equation

\[ T >> \left[ D + \mu_r (W - L) \right]_{0.7V_{LO}} \]

\[ s_{LO} = \frac{1.21(W/S)}{g \rho_a C_{T_{max}} \left\{ T/W \right\}} \]

\[ T \propto \rho_a \Rightarrow s_{LO} \propto 1/\rho_a^2 \]

\[ s_{LO} \propto W^2 \]

hot days, low air density
cold days, high air density

\( s_{LO} \) at altitude is > \( s_{LO} \) at sea level
**Distance While Airborne to Clear an Obstacle**

\[ V = \alpha V_{\text{stall}} \]

\[ C_L = 0.9 C_{L_{\text{max}}} \]

\[ n = \frac{L}{W} = \frac{\frac{1}{2} \rho \cdot S \cdot (1.15 \cdot V_{\text{stall}})^2 \cdot 0.9 \cdot C_{L_{\text{max}}}}{W} \]

\[ = \frac{1}{2} \rho \cdot S \cdot (1.15 \cdot V_{\text{stall}})^2 \cdot 0.9 \cdot C_{L_{\text{max}}} = 1.19 \]

\[ R = \frac{1.15 V_{\text{stall}}^2}{g \cdot (1.19 - 1)} = \frac{6.95 V_{\text{stall}}^2}{g} \]

\[ \cos \theta_{\text{OB}} = \frac{R - h_{\text{OB}}}{R} = 1 - \frac{h_{\text{OB}}}{R} \]

\[ s_a = R \sin \theta_{\text{OB}} \]

\[ S = S_g + s_a \]

\[ R = \frac{V_{\text{stall}}^2}{g \cdot (n-1)} \]
Landing Performance

\[ L = W \cos \theta_a \]
\[ D = T + W \sin \theta_a \]
\[ D = \frac{D - T}{W} \Rightarrow \theta_a = \frac{D}{W} - \frac{T}{W} \]
\[ \theta_a \leq 3^\circ \Rightarrow \cos \theta_a = 1 \Rightarrow L = W \]

\[ \theta_a = \frac{1}{L/W} - \frac{T}{W} \]

\[ s_a = \frac{50 - h_f}{\tan \theta_a} \]
\[ h_f = R - R \cos \theta_f = R (1 - \cos \theta_f) = R (1 - \cos \theta_a) \]

since \( \theta_f = \theta_a \)

Landing Performance

\[ R = \frac{V_{\text{ave}}^2}{g (n - 1)} \]

from \( V_a = 1.3V_{\text{stall}} \) to \( V_{TD} = 1.15V_{\text{stall}} \) commercial airplanes
from \( V_a = 1.2V_{\text{stall}} \) to \( V_{TD} = 1.1V_{\text{stall}} \) military airplanes

\[ V_{\text{ave}} = V_{\text{flare}} = 1.23V_{\text{stall}} \] commercial airplanes
\[ V_{\text{ave}} = V_{\text{flare}} = 1.15V_{\text{stall}} \] military airplanes

Load factor \( n = 1.2 \Rightarrow R = \frac{V_{\text{flare}}^2}{0.2g} \) Radius of turn

\[ s_a = \frac{50 - h_f}{\tan \theta_a} \quad \theta_a = \theta_f \quad s_f = s_{\text{flare}} = R \sin \theta_f \]
Landing data

The figure shows the landing data for two different flap angles for some airplanes. The FAR landing field length is defined as the actual demonstrated distance from a 50 ft. height to a full stop increased by the factor 1/0.60, a 67% increase.

Landing Roll – Ground Roll

- The landing roll is very similar to the takeoff ground roll, except:
  - Thrust = 0 (or near zero)
  - The sign of the acceleration is negative

\[ [D + \mu_r (W - L)]_{\text{average}} = m \frac{dV}{dt} \]

- We seek an approximate expression like \( S_{TC} \) for landing using an average force again
Landing Roll

- Following this logic
  \( F = -[D + \mu_r(W - L)]_{\text{average}} = -[D + \mu_r(W - L)]_{0.7V_T} \)
  - Notice that this assumption is less accurate for landing than for takeoff
  - Nonetheless, let’s do the integration again
  \[
  \int_{t_0}^{t_1} ds = \frac{F}{m} \int_{0}^{t_1} dt \quad \Rightarrow \quad s_L = -\frac{F}{m} \frac{t^2}{2} \quad \text{or} \quad s_L = -\frac{V^* m}{2F}
  \]

Landing Roll

- Combining the previous expression for \( s_L \) and using the approximately constant retarding force
  \[
  s_L \approx \frac{V_{TD}^2 W}{2g \left[ D + \mu_r(W - L) \right]_{0.7V_{TD}}}
  \]
  - again, adding a safety factor (10% in this case) to touch down at a speed above the stall speed
    \[
    V_{TD} \geq 1.15V_{\text{stall}} \quad \text{commercial airplanes}
    \]
    \[
    V_{TD} \geq 1.10V_{\text{stall}} \quad \text{military airplanes}
    \]
  - Substituting into the expression above for \( s_L \)
  \[
  s_L = \frac{2.645W^2}{g \rho_S C_{\text{tmax}} \left[ D + \mu_r(W - L) \right]_{0.7V_{TD}}} \quad \text{(commercial airplane)}
  \]
Landing Roll

- Landing roll can be reduced if a thrust reverser is installed

**Thrust reverser:** 40-50% max $T$ for Jet airplane; 40% max $T$ for Reciprocating engine/propeller; 60% max $T$ for Turbofan

\[-T_R - D - \mu_r (W - L) = m \frac{dV}{dt} \Rightarrow \text{where } T_R = \text{Reverse Thrust}\]

**D can be increased with:** Spoilers, speed brakes, drogue chutes

- If the thrust reverser produces constant thrust,
  \[s_L = \frac{W^2}{g \rho_{\text{air}} S C_{L_{\text{min}}} \left[ T + \left[ D + \mu_r (W - L) \right]_{0.7V_f} \right]}\]

- Lift and drag forces are calculated, accounting for ground effect as we did for takeoff roll
  \[L = \frac{1}{2} \rho_{\text{air}} V^2 C_L \quad D = \frac{1}{2} \rho_{\text{air}} V^2 S \left( C_{D,0} + \frac{C_L^2}{\pi eAR} \right)\]

Where \[G = \frac{(16h/b)^2}{1 + (16h/b)^2}\]

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**Landing Roll - sg**

\[m \frac{dV}{dt} < 0\]

\[dS = \frac{m}{2 - T_R - D - \mu_r (W - L)} \frac{d(V^2)}{dt}\]

\[s_g - s_p = \frac{W}{2g} \int_{V_{1g}}^{V_f} -T_R - D - \mu_r (W - L) \frac{d(V^2)}{dt}\]

\[s_g = NV_{TD} + \frac{W}{2g} \int_{V_{1g}}^{V_f} \frac{d(V^2)}{dt} = NV_{TD} + \frac{WV_{1g}^2}{2g} \frac{1}{T_R + D + \mu_r (W - L)}_{0.7V_f}\]

\[V_{TD} \geq 1.15V_{\text{stall}} = jV_{\text{stall}} \quad \text{commercial airplanes}\]

\[V_{TD} \geq 1.10V_{\text{stall}} = jV_{\text{stall}} \quad \text{military airplanes}\]

\[s_g = jN \frac{2}{\rho_{\text{air}} S C_{L_{\text{max}}}} + j \cdot \frac{W}{S} \frac{1}{\rho_{\text{air}} g C_{L_{\text{max}}} T_R + W + D / W + \mu_r (1 - L/W)}_{0.7V_f}\]