Lab 5
Determining the Coefficient Thermal Conductivity

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(Dated: April 18, 2010)

1. OBJECTIVES

Our objective is to analyze the behavior of thermal conductivity.

2. METHOD

When we have any material set between a high and a low temperature zone, heat energy flows from the high temperature zone to the lower, with the material in between conducting the energy. We find,

\[
\frac{dQ}{dt} = kA \frac{T' - T}{l}
\]  

(1)

Where \( \frac{dQ}{dt} \) is rate at which energy is being conducted in the material, \( A \) is the cross sectional area of the material between the high and low temperatures; \( k \) is the coefficient of thermal conductivity and \( l \) is the thickness of the material between the high and low temperature points. \( T' \) is the measure of the high temperature and \( T \) is the measure of the low temperature, to which heat is being transferred. To measure the thermal conductivity of a material, we set the high temperature to a constant, easily done by boiling water, and we place a material between the constant temperature heat source and a metallic plug. The heat conducting through the material warms the plug, so for a little heat \( dQ \) we will find a temperature increase of \( dT \).

\[
dQ = mCdT
\]  

(2)

Where \( m \) is the mass of the plug, and \( C \) is the specific heat of the plug. By dividing by \( dt \), we get:

\[
\frac{dQ}{dt} = mC \frac{dT}{dt}
\]  

(3)

and then combining with Eq(1) we form,

\[
\frac{dT}{dt} mC = kA \frac{T' - T}{l}
\]  

(4)

We now use separation of variables:

\[
dt = \frac{lmC}{kA} \frac{dT}{T' - T}
\]  

(5)

where we define \( N \) as a constant equal to,

\[
N = \frac{lmC}{kA}
\]  

(6)

Since we set \( T' \) to be a constant only \( T \) depends on time, so:

\[
\int dt = N \int \frac{dT}{T' - T}
\]

\[
t = -N \ln(T' - T) + D
\]

Where \( D \) is the constant of integration. Solving for \( \ln(T' - T) \),

\[
\ln(T' - T) = F - \frac{t}{N}
\]  

(7)

Where \( F = \frac{D}{N} \). To measure \( T' \) and \( T \) we employ a thermocouples, one for \( T' \) and one for \( T \). Here we find the net voltage to be:

\[
\varepsilon = (V' - V) = \alpha(T' - T)
\]

and if we take the natural log of both sides and combine this to Eq(7) we get,

\[
\ln(\varepsilon) = \ln(\alpha) + F - \frac{t}{N}
\]

After combining constants,

\[
\ln(\varepsilon) = F' - \frac{t}{N}
\]  

(8)

**Determining the Coefficient of Thermal Conductivity, \( k \)**

To determine \( k \) we measured the value of \( \varepsilon \) at 1.5min intervals. After collecting data and plotting the \( \ln(\varepsilon) \) vs. \( t \), we can find the best fit line giving us a slope. and while examining Eq(8) we find,

\[
Slope = -\frac{1}{N} = \frac{kA}{lmC}
\]  

(9)

So,

\[
k = \frac{(Slope)mlC}{A}
\]  

(10)
3. DATA AND RESULTS

Question 1/2

TABLE I: Effective $\varepsilon$ over time, $t$

<table>
<thead>
<tr>
<th>$t$(min)</th>
<th>$t$(s)</th>
<th>$\varepsilon$(V)</th>
<th>$\ln(\varepsilon)$</th>
</tr>
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</tr>
<tr>
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<td>90</td>
<td>0.00297</td>
<td>-5.8191</td>
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<td>2160</td>
<td>0.00216</td>
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\[ y = -0.0002x - 5.8072 \]
\[ R^2 = 0.99785 \]

$\text{Plot 1: Data Plot of } \ln(E) \text{ vs. } t$

Question 3

We calculate the average $\text{Area} = 15.3717 \text{cm}^2$, and the average thickness $L = 0.496 \text{cm}$

Question 4

Since,

$$k = \frac{\text{(Slope)} \cdot mlC}{A}$$

We find $k = 0.00020252 \frac{\text{cal}}{\text{s} \cdot \text{cm} \cdot ^\circ \text{C}} = 0.0848 \frac{\text{W}}{\text{mK}}$

An excepted value of $k_{\text{cork}} = 0.07 \frac{\text{W}}{\text{mK}} [1]$

Giving us a percent difference of 21.23%.

Question 5

In the working equation, Eq(1), we require that the area of the sample be greater than or equal to that of the plug. If we had a sample which did not fully cover the surface of the plug, we then would be measuring the effective $k$. Because we then would have two different mediums through which the heat would be traveling, through air and our sample. So, we strictly require that $r_{\text{sample}} \geq r_{\text{plug}}$.

Question 6

We find the $k$ values of wood to be $k_{\text{wood}} \approx 0.04 \frac{\text{W}}{\text{mK}}$ and metal (an average of copper, gold, silver and aluminum) $k \approx 200 \frac{\text{W}}{\text{mK}}$. When we put our hand on the metal, heat can very readily transfer from our hand to the metal, while with the wood the heat can not transfer as fast. Since the heat is transferring from the hand to the material, the hand feels cooler due to the loss of heat in itself. The difference in the felt temperature between the metal and the wood, comes from the fact that the $k$ values vary greatly, by a factor of $\approx 10^4$.

Question 7

If we are using the same dimensions, we can draw from this that the volume, $V$, stays constant. If we use a different sample material we can assume that the mass density changes to, $\rho_{\text{new}}$. Since mass, $m = V \rho_{\text{new}}$, we can say that $m$ changes. Since we are working with a different material, our specific heat, $C$, also changes. We know $B = \frac{kA}{mlC}$, Eq(9), so $B$ would also change due the change of material.
Question 8

Given: \( T_c = -1.11^\circ C, T_h = 20^\circ C, L = .3cm, \) and \( k_{glass} = .01 \frac{W}{cmK} \) So the power per unit area is,

\[
P = \frac{k(T_h - T_c)}{L}
\]

giving us \( .70366 \frac{W}{cm^2} \)

Question 9

Given: \( l = 5cm, T_c = -10^\circ C, \rho = .92 \frac{g}{cm^3}, k = .004 \frac{cal}{s.cmK}, L_f = 79.5 \frac{cal}{g} \) and we assume the water is at freezing, \( T_h = 0^\circ C. \)

Rewriting the energy of formation in differential form we get,

\[
dQ = dm L_f
\]

From taking the time derivative of the mass density, where \( m(t) \) and \( l(t) \), we find

\[
\frac{dm}{dt} = \frac{dl}{dt} \rho
\]

Using the previous equation we can get \( \frac{dl}{dt} \) in terms of the power,

\[
\frac{dQ}{dt} = \frac{dl}{dt} \rho L_f
\]

We now can use Eq(1) to equate \( \frac{dl}{dt} \) with the change in temperature.

\[
\frac{dl}{dt} = \frac{k(T_h - T_c)}{\rho L_f l}
\]

Giving us \( \frac{dl}{dt} = .39376 \frac{cm}{s} \)

Question 10

Given: \( A = 1.8m^2, L = 0.01m, T_h = 33^\circ C, T_c 1.0^\circ C \)

\[ k_{dry} = 0.04 \frac{W}{mK} \] and \[ k_{wet} = 0.6 \frac{W}{mK} \]

\[ a. \]

Using,

\[
P = \frac{k_{dry}A(T_h - T_c)}{L}
\]

We find \( P = 230.4W \)

\[ b. \]

Now using \( k_{wet} \) we find \( P = 3.46kW \)

Question 11

We are given two materials of \( k_1 \) and \( k_2 \) of thickness \( L \). We are to find the arrangement which yield the lower energy flow.

\[ a. \]

\[
P_a = P_1 + P_2
\]

\[
= \frac{A(T_h - T_c)}{\frac{k_1}{L}} + \frac{A(T_h - T_c)}{\frac{k_2}{L}}
\]

\[
= \frac{A(T_h - T_c)}{L} \left( \frac{k_1 + k_2}{2} \right)
\]

Where \( \frac{k_1 + k_2}{2} \) is the effective \( k_a \).

\[ b. \]

\[
P_b = P_1 + P_2
\]

\[
= \frac{2A(T_h - T_c)}{\frac{k_1}{L} + \frac{k_2}{L}}
\]

\[
= \frac{A(T_h - T_c)}{L} \left( \frac{2k_1k_2}{k_1 + k_2} \right)
\]

Where \( \frac{2k_1k_2}{k_1 + k_2} \) is the effective \( k_b \).

Now we want to find the ratio between \( k_a \) and \( k_b \)

\[
\frac{k_a}{k_b} = \frac{\frac{k_1 + k_2}{2}}{\frac{k_1 + k_2}{k_1 + k_2}}
\]

\[
= \frac{(k_1 + k_2)^2}{4k_1k_2}
\]

On examining this relationship we see that for all \( k > 0 \),

\[
\frac{(k_1 + k_2)^2}{4k_1k_2} > 1
\]

Thus saying, \( \frac{k_a}{k_b} > 1 \), and then \( k_a > k_b \). So, arrangement \( b \) has the lower flow.
Question 12

Given: \( k_1 = 0.06 \frac{W}{mK} \), \( k_4 = 0.12 \frac{W}{mK} \), \( L_1 = 0.015 m \), \( L_4 = 0.035 m \), \( P_1 = P_2 = \cdots = P_i \), \( A_1 = A_2 = \cdots = A_i \), \( T_h = 30^\circ C \), \( T_2 = 25^\circ C \) and \( T_c = -10^\circ C \).

Find \( T_4 \).

So, we can say:

\[
\frac{k_1 A (T_h - T_2)}{L_1} = \frac{k_4 A (T_4 - T_c)}{L_4}
\]

Now we solve for \( T_4 \),

\[
T_4 = \frac{k_1 L_2}{k_1 L_1} (T_h - T_2) + T_c
\]

Giving us \( T_4 = -4.2^\circ C \)

Discussion

On examining the dependencies of \( k \) on measured values we find certain sources of error. First with the errors associated with slope, we find that we measured \( \varepsilon \) at a very specific interval of time, but this was accomplished by hand so there was a slight delay in the determination of \( \varepsilon \). Now looking at Eq(10), we are given \( m \) and \( C \) as constants, but \( l \) and \( A \) are not. the error associated with \( l \) is quite significant, because the surface of the material is not uniform, it is a loose composite material. Measuring \( A \) is also inaccurate, since we cannot fully fall on the diameter of the circle. However, one of the largest errors is the material itself. Since the material is a loose composite, we are not getting the \( k \) for cork, but for cork and air. This could explain, If we assume all other errors were very small, that the material has a higher \( k \) value when compared to the excepted value[1].

Overall, this experiment produced results that fell close to that of the excepted value.