Voltage and Current Division

Introduction

The circuits in this problem set consist of a single voltage or current source, some resistors and a voltmeter or ammeter. The input to each circuit is the voltage of the voltage source or the current of the current source. The output of each circuit is either the voltage measured by the voltmeter or the current measured by the ammeter. All of the inputs have constant values. Consequently, the outputs also have constant values. The output of each circuit is proportional to the input to that circuit.

Each circuit includes resistors that are connected either in series or in parallel. The problems in this problem set can be solved using the principles of voltage division, current division or equivalent resistance.

Series resistors and voltage division are discussed in Section 3.4 of *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda. Parallel resistors and current division are discussed in Section 3.4. These ideas are summarized in Table 3.12-1 of *Introduction to Electric Circuits*.

Worked Examples

Example 1:
Consider the voltage divider circuit shown in Figure 1. Find the value of the resistance \( R \). Determine the power supplied by the voltage source.

![Figure 1](image)

**Figure 1** The circuit considered in Example 1.

**Solution:** The voltmeter measures the voltage across the 4 \( \Omega \) resistor to be –6 V. (The color coded probes of the voltmeter indicate the reference direction of the voltage measured by the voltmeter.) Figure 2a shows the circuit after the replacing the voltmeter by the equivalent open circuit and adding a label to show the voltage measured by the meter.

Figure 2b shows the voltage divider circuit again, this time with the voltage across the 4 \( \Omega \) resistor labeled as \( v_a \). The voltage division equation gives
\[ v_a = \left( \frac{4}{4+R} \right) 15 \]  

(1)

The reference direction for \( v_a \), + on the top, is the opposite of the reference direction for the voltage measured by the voltmeter, + on the bottom. Thus \( v_a = -(-6) = 6 \) V. Substituting this value into equation 1 gives

\[ 6 = \left( \frac{4}{4+R} \right) 15 \implies R = 6 \Omega \]

In Figure 2c the currents in the voltage source and the two resistors have been labeled. With the reference directions selected as shown in Figure 2c, these three currents are equal. Consequently, all three currents have been given the same name, \( i \). The value of the current \( i \) can be determined by applying Ohm’s Law to the 4 \( \Omega \) resistor:

\[ i = \frac{-6}{4} = -1.5 \text{ A} \]

The current \( i \) and voltage 15 V, of the voltage source adhere to the passive convention. The product of this current and voltage is the power received by the voltage source. The power supplied by the voltage source is the negative of the power received by the voltage source. The power supplied by the voltage source is \((-1.5)(15) = 22.5 \text{ W} \).

\[ \text{(a)} \quad \text{The circuit of Figure 1 after replacing the voltmeter by the equivalent open circuit.} \]

\[ \text{(b)} \quad \text{The circuit labeled differently.} \]

\[ \text{(c)} \quad \text{The circuit of Figure 1 after labeling the element currents.} \]
Example 2:
Consider the current divider circuit shown in Figure 3. Find the value of the resistance \( R \).
Determine the power supplied by the current source.

![Figure 3 The circuit considered in Example 2.](image)

**Solution:** The ammeter measures the current in the 40 Ω resistor to be –0.4 A. (The color coded probes of the ammeter indicate the reference direction of the current measured by the ammeter.) Figure 4a shows the circuit after the replacing the ammeter by the equivalent short circuit and adding a label to show the current measured by the meter.

Figure 4b shows the current divider circuit again, this time with the current the 40 Ω resistor labeled as \( i_a \). The current division equation gives

\[
i_a = \left( \frac{R}{R + 40} \right)^2 \tag{2}
\]

The reference direction for \( i_a \), downward, is the opposite of the reference direction for the current measured by the ammeter, upward. Thus \( i_a = -(-0.4) = 0.4 \) A. Substituting this value into equation 2 gives

\[
0.4 = \left( \frac{R}{R + 40} \right)^2 \Rightarrow R = 10 \Omega
\]

In Figure 4c the voltages across the current source and the two resistors have been labeled. With the reference directions selected as shown in Figure 4c, these three voltages are equal. Consequently, all three voltages have been given the same name, \( v \). The value of the voltage \( v \) can be determined by applying Ohm’s Law to the 40 Ω resistor:

\[
v = (-0.4)(40) = -16 \text{ V}
\]

The voltage \( v \), and current 2 A, of the current source adhere to the passive convention. The product of this current and voltage is the power received by the current source. The power supplied by the current source is the negative of the power received by the current source. The power supplied by the current source is \(-2 \times -16 = 32 \text{ W}\).
Figure 4 (a) The circuit of Figure 3 after replacing the ammeter by the equivalent short circuit. (b) The circuit labeled differently. (c) The circuit of Figure 3 after labeling the element voltages.
Example 3:
Consider the circuit shown in Figure 5. Find the value of the resistance $R$.

![Figure 5](image)

**Figure 5** The circuit considered in Example 3.

Solution: The ammeter measures the current in the series resistors to be $-2$ A. (The color coded probes of the ammeter indicate the reference direction of the current measured by the ammeter.) Figure 6 shows the circuit after the replacing the series resistors by a single equivalent resistor, replacing the ammeter by an equivalent short circuit and adding a label to show the current measured by the meter.

![Figure 6](image)

**Figure 6** A circuit that is equivalent to the circuit of Figure 5.

The equivalent resistance is given by the sum of the series resistances:

$$R_{eq} = R + 6 \quad (3)$$

Applying KVL to the single mesh circuit in Figure 6 gives

$$-R_{eq} (-2) - 30 = 0 \quad \Rightarrow \quad R_{eq} = 15 \Omega$$

Substituting this value into Equation 3 gives $R = 9 \Omega$. 
Example 4:
Consider the circuit shown in Figure 7. Find the value of the resistance $R$.

![Figure 7 The circuit considered in Example 4.](image)

Solution: The voltmeter measures the voltage across the parallel resistors to be $-24$ V. (The color coded probes of the voltmeter indicate the reference direction of the voltage measured by the voltmeter.) Figure 8 shows the circuit after the replacing the parallel resistors by a single equivalent resistor, replacing the voltmeter by an equivalent open circuit and adding a label to show the voltage measured by the meter.

![Figure 8 A circuit that is equivalent to the circuit of Figure 7.](image)

The equivalent resistance is given by

$$R_{eq} = \frac{R(40)}{R + 40} \quad (4)$$

Applying KCL at the top node of the circuit in Figure 8 gives

$$3 + \frac{-24}{R_{eq}} = 0 \quad \Rightarrow \quad R_{eq} = 8 \, \Omega$$

Substituting this value into Equation 4 gives

$$8 = \frac{R(40)}{R + 40} \quad \Rightarrow \quad 8R + (8)(40) = 40R \quad \Rightarrow \quad R = 10 \, \Omega$$
Example 5:
Consider the circuit shown in Figure 9. Find the values of the resistance $R$ and of the current, $i$.

Solution: See Example 3.4-1 in *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda.

Example 6:
Consider the circuit shown in Figure 10. Find the value of the current measured by the meter. Show that the sum of the power absorbed by the two resistors is equal to the power supplied by the voltage source.

Solution: See Example 3.4-2 in *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda.
Example 7:
Consider the circuit shown in Figure 11. Find the value of the voltage measured by the meter. Show that the sum of the power absorbed by the two resistors is equal to the power supplied by the current source.

![Circuit Diagram](image)

**Figure 11** The circuit considered in Example 7.

**Solution:** See Example 3.5-2 in *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda.