Equivalent Circuits

Introduction

The circuits in this set of problems consist of a voltage or current source and several resistors. The resistors are connected together to form a “resistor sub-circuit”. These circuits can be simplified by repeatedly replacing series or parallel resistors by an equivalent resistor. Eventually, the resistor sub-circuit is reduced to a single equivalent resistor.

In each problem we are asked to determine the values of three currents or voltages. These currents or voltages are identified by the subscripts a, b and c. The computer will guide us to a solution in three steps:

1. Reduce the resistor sub-circuit to a single resistor.

2. Analyze the reduced circuit, using Ohm’s law, to find the resistor current and voltage. Then determine the values of the source current and voltage in the reduced circuit. The values of the source current and voltage in the original circuit are the same as the values of the source current and voltage in the reduced circuit.

3. Complete the analysis of the original circuit using voltage or current division.

Series resistors are discussed in Section 3.4 of Introduction to Electric Circuits by R.C. Dorf and J.A Svoboda. Parallel resistors are discussed in Section 3.5. Circuit analysis using equivalent resistances is described in Section 3.7.

Worked Examples

Example 1:
Consider the circuit shown in Figure 1. Find the values of the voltage $v_c$ and the current $i_b$.

![Figure 1 The circuit considered in Example 1.](image)
**Solution:** The terminals in Figure 1 divide the circuit into two parts, the part to the left of the terminals and the part to the right of the terminals. The part to the right of the terminals consists of three resistors. The 10 Ω resistor is connected in parallel with the 40 Ω resistor and that parallel combination is connected in series with the 4 Ω resistor. These three resistors can be replaced by a single equivalent resistor as shown in Figure 2a. The resistance of the equivalent resistor is given by

\[
R_{eq} = 4 + \frac{(10)(40)}{10 + 40} = 4 + \frac{800}{50} = 4 + 16 = 20 \Omega
\]

The current in the equivalent resistance is determined using Ohm’s law to be

\[
i_a = \frac{24}{12} = 2 \text{ A}
\]

The values of the equivalent resistance and the current \(i_a\) are labeled in Figure 2b.

The circuit shown in Figure 2b is equivalent to the circuit shown in Figure 1. Consequently, the value of the current \(i_a\) in Figure 1 is equal to the value of the current \(i_a\) in Figure 2b. Figure 2c shows the circuit from Figure 1 after labeling the value of the current \(i_a\). Now the voltage \(v_c\) can be calculated using Ohm’s law to be

\[
v_c = (2)(4) = 8 \text{ V}
\]

The current \(i_b\) can be calculated using current division to be

\[
i_b = -\frac{10}{10 + 40}(2) = -0.4 \text{ A}
\]

Figure 2d shows the circuit from Figure 1 after labeling the values of the voltage \(v_c\) and the current \(i_b\).
Figure 2 (a) The circuit of Figure 1 after replacing the resistors by a single equivalent resistor. (b) The values of the equivalent resistance and the current $i_a$. (c) The circuit of Figure 1 after labeling the value of the current $i_a$. (d) The values of the voltage $v_c$ and the current $i_b$. 
Example 2:
Consider the circuit shown in Figure 3. Find the values of the voltage $v_c$ and the current $i_b$.

![Figure 3](image-url) The circuit considered in Example 2.

Solution: The terminals in Figure 3 divide the circuit into two parts, the part to the left of the terminals and the part to the right of the terminals. The part to the right of the terminals consists of three resistors. The 4 Ω resistor is connected in series with the 2 Ω resistor and that series combination is connected in parallel with the 3 Ω resistor. These three resistors can be replaced by a single equivalent resistor as shown in Figure 4a. The resistance of the equivalent resistor is given by

$$R_{eq} = \frac{(3)(4+2)}{3+(4+2)} = \frac{18}{9} = 2 \Omega$$

The voltage across the equivalent resistance is determined using Ohm’s law to be

$$v_a = (6)(2) = 6 \text{ V}$$

The values of the equivalent resistance and the voltage $v_a$ are labeled in Figure 4b. The circuit shown in Figure 4b is equivalent to the circuit shown in Figure 3. Consequently, the value of the voltage $v_a$ in Figure 3 is equal to the value of the voltage $v_a$ in Figure 4b. Figure 4c shows the circuit from Figure 3 after labeling the value of the voltage $v_a$. Now the current $i_b$ can be calculated using Ohm’s law to be

$$i_b = \frac{6}{3} = 2 \text{ A}$$

The voltage $v_c$ can be calculated using voltage division to be

$$v_c = -(\frac{4}{4+2})(6) = -4 \text{ V}$$
Figure 4d shows the circuit from Figure 3 after labeling the values of the voltage $v_c$ and the current $i_b$.

**Figure 4** (a) The circuit of Figure 3 after replacing the resistors by a single equivalent resistor. (b) The values of the equivalent resistance and the voltage $v_a$. (c) The circuit of Figure 3 after labeling the value of the voltage $v_a$. (d) The values of the voltage $v_c$ and the current $i_b$. 
Example 3:
Consider the circuit shown in Figure 5. Find the values of the voltage $v_c$ and the current $i_b$.

![Figure 5 The circuit considered in Example 3.](image)

Solution: The terminals in Figure 5 divide the circuit into two parts, the part to the left of the terminals and the part to the right of the terminals. The part to the right of the terminals consists of three resistors. A 10 Ω resistor is connected in series with the 30 Ω resistor and that series combination is connected in parallel with the other 10 Ω resistor. These three resistors can be replaced by a single equivalent resistor as shown in Figure 6a. The resistance of the equivalent resistor is given by

$$R_{eq} = \frac{(10)(30+10)}{10 + (30+10)} = \frac{400}{50} = 8 \, \Omega$$

The current in the equivalent resistance is determined using Ohm’s law to be

$$i_a = \frac{40}{8} = 5 \, \text{A}$$

The values of the equivalent resistance and the current $i_a$ are labeled in Figure 6b. The circuit shown in Figure 6b is equivalent to the circuit shown in Figure 5. Consequently, the value of the current $i_a$ in Figure 5 is equal to the value of the current $i_a$ in Figure 6b. Figure 6c shows the circuit from Figure 5 after labeling the value of the current $i_a$.

Consider the 10 Ω resistor having current $i_b$. The voltage across this resistor is 40 V. Notice that the current $i_b$ and the voltage 40 V do not adhere to the passive convention. The current $i_b$ can be calculated using Ohm’s law to be

$$i_b = -\frac{40}{10} = -4 \, \text{A}$$

The voltage $v_c$ can be calculated using voltage division to be...
$$v_c = \left(\frac{30}{30 + 10}\right)(40) = 30 \text{ V}$$

Figure 6d shows the circuit from Figure 5 after labeling the values of the voltage $v_c$ and the current $i_b$.

Figure 6  
(a) The circuit of Figure 5 after replacing the resistors by a single equivalent resistor.  
(b) The values of the equivalent resistance and the current $i_a$.  
(c) The circuit of Figure 5 after labeling the value of the current $i_a$.  
(d) The values of the voltage $v_c$ and the current $i_b$.  

7
Example 4:
Consider the circuit shown in Figure 7. Find the values of the voltage $v_c$ and the current $i_b$.

Figure 7 The circuit considered in Example 4.

Solution: The terminals in Figure 7 divide the circuit into two parts, the part to the left of the terminals and the part to the right of the terminals. The part to the right of the terminals consists of three resistors. The $3 \Omega$ resistor is connected in parallel with the $6 \Omega$ resistor and that parallel combination is connected in series with the $4 \Omega$ resistor. These three resistors can be replaced by a single equivalent resistor as shown in Figure 8a. The resistance of the equivalent resistor is given by

$$R_{eq} = 4 + \frac{(6)(3)}{6+3} = 4 + 2 = 6 \Omega$$

The voltage across the equivalent resistor is $v_a$ and the value of the current in the equivalent resistor is 6 A. Notice that this current and voltage do not adhere to the passive convention. The voltage across the equivalent resistance is determined using Ohm’s law to be

$$v_a = -(6)(6) = -36 \text{ V}$$

The values of the equivalent resistance and the voltage $v_a$ are labeled in Figure 8b.

The circuit shown in Figure 8b is equivalent to the circuit shown in Figure 7. Consequently, the value of the voltage $v_a$ in Figure 7 is equal to the value of the voltage $v_a$ in Figure 8b. Figure 8c shows the circuit from Figure 7 after labeling the value of the voltage $v_a$.

The voltage across the $4 \Omega$ resistor is $v_c$ and the value of the current in the $4 \Omega$ resistor is 6 A. Notice that this current and voltage do not adhere to the passive convention. The voltage $v_c$ can be calculated using Ohm’s law to be

$$v_c = -(6)(4) = 24 \text{ V}$$

The current $i_b$ can be calculated using current division to be
\[ i_b = -\left( \frac{6}{6+3} \right) (6) = -4 \text{ A} \]

Figure 8d shows the circuit from Figure 7 after labeling the values of the voltage \( v_c \) and the current \( i_b \).

**Figure 8**

(a) The circuit of Figure 7 after replacing the resistors by a single equivalent resistor.

(b) The values of the equivalent resistance and the voltage \( v_a \).

(c) The circuit of Figure 7 after labeling the value of the voltage \( v_a \).

(d) The values of the voltage \( v_c \) and the current \( i_b \).