AC Circuits with Coupled Inductors

Introduction

The circuits in this problem set contain coupled inductors. Each problem involves the steady state response of such a circuit to a single sinusoidal input. That input is either the voltage of an independent voltage source or the current of an independent current source.

Circuit analysis in the frequency-domain provides the solutions to these problems.

Coupled inductors are described in Section 11.9 of Introduction to Electric Circuits by R.C. Dorf and J.A Svoboda. In particular, Table 11.14-1 summarizes the equations that represent coupled inductors in the frequency domain. Circuit analysis in the frequency domain is described in Sections 10.6 thru 10.11. Table 10.7-1 summarizes the correspondence between the time domain and the frequency domain.

Worked Examples

Example 1:
Consider the circuit shown in Figure 1. The input to the circuit is the current of the current source, $i_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$.

\[ i_s(t) = 707 \cos (5t + 47^\circ) \text{ mA} \]

![Figure 1](image.png)  
**Figure 1** The circuit considered in Example 1.

**Solution:** The input current is a sinusoid and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input current. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances. Figure 2 shows the frequency domain representation of the circuit from Figure 1.
The circuit in Figure 2 consists of a single mesh. The mesh current is equal to the current source $I_s(\omega)$. The current $I_s(\omega)$ enters the dotted end of both coils, consequently the voltage across the right hand coil is given by

$$V_x(\omega) = j10I_s(\omega) + j5I_s(\omega) = j15I_s(\omega) = j15(0.707 \angle 47^\circ)$$

$$= (15 \angle 90^\circ)(0.707 \angle 47^\circ)$$

$$= (15)(0.707) \angle (90 + 47)^\circ$$

$$= 10.605 \angle 137^\circ \text{ V}$$

In the time domain, the output voltage is given by

$$v_o(t) = 10.6 \cos(5t + 137^\circ) \text{ V}$$

Example 2:
Consider the circuit shown in Figure 3. The input to the circuit is the current of the current source, $i_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$.

$$i_s(t) = 707 \cos(5t + 47^\circ) \text{ mA}$$

**Solution:** The circuit shown in Figure 3 is very similar to the circuit shown in Figure 1. There is only one difference: the dot of the right-hand coil is located at the bottom of the coil in Figure 3 and at the top of the coil in Figure 1. As in Example 1, our first step is to represent the circuit in
the frequency domain, using phasors and impedances. Figure 4 shows the frequency domain representation of the circuit from Figure 3.

![Figure 4](image)

**Figure 4** The circuit from Figure 3, represented in the frequency domain, using impedances and phasors.

The circuit in Figure 4 consists of a single mesh. The mesh current is equal to the current source $I_s(\omega)$. The current $I_s(\omega)$ enters the dotted end of the left-hand coil and enters the undotted end of the right-hand coil. Consequently the voltage across the right-hand coil is given by

$$V_o(\omega) = j20$$

$$I_s(\omega) = 0.707 \angle 47^\circ \text{ A}$$

$$I_s(\omega) = j5$$

$$V_o(\omega)$$

$$V_o = 3.535 \angle 137^\circ \text{ V}$$

In the time domain, the output voltage is given by

$$v_o(t) = 3.535 \cos (5t + 137^\circ) \text{ V}$$

**Example 3:**
Consider the circuit shown in Figure 5. The input to the circuit is the current of the current source, $i_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$.

![Figure 5](image)

**Figure 5** The circuit considered in Example 3.
**Solution:** The input current is sinusoid and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input current. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances. Figure 6 shows the frequency domain representation of the circuit from Figure 5.

![Figure 6](image)

Figure 6 The circuit from Figure 5, represented in the frequency domain, using impedances and phasors.

The coil currents, \( I_1(\omega) \) and \( I_2(\omega) \), and the coil voltages, \( V_1(\omega) \) and \( V_2(\omega) \), are labeled in Figure 6. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that one coil current, \( I_1(\omega) \), enters the undotted end of its coil while the other coil current, \( I_2(\omega) \), enters the dotted end of its coil.

The device equations for coupled coils are:

\[
V_1(\omega) = j16 \, I_1(\omega) - j8 \, I_2(\omega) \tag{1}
\]

and

\[
V_2(\omega) = -j8 \, I_1(\omega) + j12 \, I_2(\omega) \tag{2}
\]

The coils are connected in parallel, consequently \( V_1(\omega) = V_2(\omega) \). Equating the expressions for \( V_1(\omega) \) and \( V_2(\omega) \) in Equations 1 and 2 gives

\[
j16 \, I_1(\omega) - j8 \, I_2(\omega) = -j8 \, I_1(\omega) + j12 \, I_2(\omega)
\]

\[
j24 \, I_1(\omega) = j20 \, I_2(\omega)
\]

\[
I_1(\omega) = \frac{5}{6} \, I_2(\omega)
\]

Apply Kirchhoff’s Current Law (KCL) to the top node of the coils to get
\[
0.121\angle140^\circ = I_1(\omega) = I_1(\omega) + I_2(\omega) = \frac{5}{6} I_2(\omega) + I_2(\omega) = \frac{11}{6} I_2(\omega)
\]

\[
I_2(\omega) = \frac{6}{11}(0.121\angle140^\circ) = 0.066\angle140^\circ \text{ A}
\]

Now the output voltage can be calculated using Equation 2:

\[
V_o(\omega) = V_2(\omega) = -j 8 I_1(\omega) + j 12 I_2(\omega)
\]

\[
= -j 8 \left( \frac{5}{6} I_2(\omega) \right) + j 12 I_2(\omega) = j \frac{-8(5) + 12(6)}{6} I_2(\omega)
\]

\[
= j 5.333 I_2(\omega)
\]

\[
= (5.333\angle90^\circ)(0.066\angle140^\circ) = 0.352\angle230^\circ \text{ V}
\]

In the time domain, the output voltage is given by

\[
v_o(t) = 0.352 \cos(4t + 230^\circ) \text{ V}
\]

**Example 4:**

Consider the circuit shown in Figure 7. The input to the circuit is the current of the current source, \(i_s(t)\). The output is the voltage across the right-hand coil, \(v_o(t)\). Determine the steady-state output voltage, \(v_o(t)\).

![Figure 7](image)

**Figure 7** The circuit considered in Example 4.

**Solution:** The circuit shown in Figure 7 is very similar to the circuit shown in Figure 5. There is only one difference: the dot of the left-hand coil is located at the bottom of the coil in Figure 5 and at the top of the coil in Figure 7. As in Example 3, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 8 shows the frequency domain representation of the circuit from Figure 7.
The coil currents, $I_1(\omega)$ and $I_2(\omega)$, and the coil voltages, $V_1(\omega)$ and $V_2(\omega)$, are labeled in Figure 8. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that the coil currents, $I_1(\omega)$ and $I_2(\omega)$, both enter the dotted of their respective coils.

The device equations for coupled coils are:

$$V_1(\omega) = j16 I_1(\omega) + j8 I_2(\omega) \quad (3)$$

and

$$V_2(\omega) = j8 I_1(\omega) + j12 I_2(\omega) \quad (4)$$

The coils are connected in parallel, consequently $V_1(\omega) = V_2(\omega)$. Equating the expressions for $V_1(\omega)$ and $V_2(\omega)$ in Equations 3 and 4 gives

$$j16 I_1(\omega) + j8 I_2(\omega) = j8 I_1(\omega) + j12 I_2(\omega)$$

$$j8 I_1(\omega) = j4 I_2(\omega)$$

$$I_1(\omega) = \frac{1}{2} I_2(\omega)$$

Apply Kirchhoff’s Current Law (KCL) to the top node of the coils to get

$$0.121 \angle 140^\circ = I_s(\omega) = I_1(\omega) + I_2(\omega) = \frac{1}{2} I_2(\omega) + I_2(\omega) = \frac{3}{2} I_2(\omega)$$

$$I_2(\omega) = \frac{2}{3} (0.121 \angle 140^\circ) = 0.08067 \angle 140^\circ \text{ A}$$

Now the output voltage can be calculated using Equation 4:
\[ V_o(\omega) = V_z(\omega) = j81 I_1(\omega) + j12 I_2(\omega) \]
\[ = j81 \left( \frac{1}{2} I_2(\omega) \right) + j12 I_2(\omega) \]
\[ = j16 I_2(\omega) \]
\[ = (16\angle 90^\circ)(0.04033\angle 140^\circ) = 1.291\angle 230^\circ \text{ V} \]

In the time domain, the output voltage is given by
\[ v_o(t) = 1.291 \cos(4t + 230^\circ) \text{ V} \]

**Example 5:**
Consider the circuit shown in Figure 9. The input to the circuit is the voltage of the voltage source, \( v_s(t) \). The output is the voltage across the right-hand coil, \( v_o(t) \). Determine the steady-state output voltage, \( v_o(t) \).

**Figure 9** The circuit considered in Example 5.

**Solution:** The input voltage is a sinusoid and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input voltage. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances. Figure 10 shows the frequency domain representation of the circuit from Figure 9.

**Figure 10** The circuit from Figure 9, represented in the frequency domain, using impedances and phasors.
Notice that the current, \( I_2(\omega) \), in the right-hand coil is zero and that both coil currents, \( I_1(\omega) \) and \( I_2(\omega) \), enter the undotted ends of their respective coils. The voltage, \( V_1(\omega) \), across the left-hand coil is given by

\[
V_1(\omega) = j \cdot 25 I_1(\omega) + j \cdot 15 (0) = j \cdot 25 I_1(\omega)
\]

(5)

Apply KVL to the mesh consisting of the voltage source, resistor and the left-hand coil to get

\[
8 I_1(\omega) + V_1(\omega) - 7.32\angle95^\circ = 0
\]

Substituting the expression for \( V_1(\omega) \) from Equation 5 gives

\[
8 I_1(\omega) + j \cdot 25 I_1(\omega) - 7.32\angle95^\circ = 0
\]

Now solve for \( I_1(\omega) \) to get

\[
I_1(\omega) = \frac{7.32\angle95^\circ}{8 + j \cdot 25} = \frac{7.32\angle95^\circ}{26.25\angle72^\circ} = 0.279\angle23^\circ \text{ V}
\]

The voltage, \( V_o(\omega) \), across the right-hand coil is given by

\[
V_o(\omega) = j \cdot 15 I_1(\omega) + j \cdot 10 (0) = j \cdot 15 I_1(\omega) = j \cdot 15 (0.279\angle23^\circ) = (15\angle90^\circ)(0.279\angle23^\circ) = 4.185\angle113^\circ \text{ V}
\]

In the time domain, the output voltage is given by

\[
v_o(t) = 4.185 \cos(4t + 113^\circ) \text{ V}
\]
Example 6:
Consider the circuit shown in Figure 11. The input to the circuit is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$.

![Figure 11 The circuit considered in Example 6.](image1)

Solution: The circuit shown in Figure 11 is very similar to the circuit shown in Figure 9. There is only one difference: the dot of the right-hand coil is located at the bottom of the coil in Figure 9 and at the top of the coil in Figure 11. As in Example 5, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 12 shows the frequency domain representation of the circuit from Figure 11.

![Figure 12 The circuit from Figure 11, represented in the frequency domain, using impedances and phasors.](image2)

Notice that the current, $I_2(\omega)$, in the right-hand coil is zero and that one of coil currents, $I_1(\omega)$, enters the undotted end of the coil while the other coil current, $I_2(\omega)$, enters the dotted end of the coil. The voltage, $V_1(\omega)$, across the left-hand coil is given by

$$V_1(\omega) = j \omega I_1(\omega) - j \omega (0) = j \omega I_1(\omega)$$

(6)

Apply KVL to the mesh consisting of the voltage source, resistor and the left-hand coil to get

$$8 I_1(\omega) + V_1(\omega) - 7.32 \angle 95^\circ = 0$$
Substituting the expression for $V_1(\omega)$ from Equation 6 gives

$$8I_1(\omega) + j25I_1(\omega) - 7.32 \angle 95^\circ = 0$$

Now solve for $I_1(\omega)$ to get

$$I_1(\omega) = \frac{7.32 \angle 95^\circ}{8 + j25} = \frac{7.32 \angle 95^\circ}{26.25 \angle 72^\circ} = 0.279 \angle 23^\circ \text{ V}$$

The voltage, $V_\omega(\omega)$, across the right-hand coil is given by

$$V_\omega(\omega) = -j15I_1(\omega) + j10(0) = -j15I_1(\omega) = -j15(0.279 \angle 23^\circ)$$

$$= (15 \angle -90^\circ)(0.279 \angle 23^\circ) = 4.185 \angle -67^\circ \text{ V}$$

In the time domain, the output voltage is given by

$$v_o(t) = 4.185 \cos(4t - 67^\circ) \text{ V}$$

**Example 7:**

Consider the circuit shown in Figure 13. The input to the circuit is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$.

**Solution:** The input voltage is a sinusoid and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input voltage. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances. Figure 14 shows the frequency domain representation of the circuit from Figure 13.
Figure 14 The circuit from Figure 13, represented in the frequency domain, using impedances and phasors.

The circuit in Figure 14 consists of a single mesh. Notice that the mesh current, $I(\omega)$, enters the undotted ends of both coils. Apply KVL to the mesh to get

$$5I(\omega) + (j12I(\omega) + j6I(\omega)) + (j6I(\omega) + j15I(\omega)) - 5.94\angle140^\circ = 0$$

$$5I(\omega) + (j12 + j6 + j6 + j15)I(\omega) - 5.94\angle140^\circ = 0$$

$$I(\omega) = \frac{5.94\angle140^\circ}{5 + j(12 + 6 + 6 + 15)} = \frac{5.94\angle140^\circ}{5 + j39} = \frac{5.94\angle140^\circ}{39.3\angle83} = 0.151\angle57^\circ \ A$$

Notice that the voltage, $V_o(\omega)$, across the right-hand coil and the mesh current, $I(\omega)$, adhere to the passive convention. The voltage across the right-hand coil is given by

$$V_o(\omega) = j15I(\omega) + j6I(\omega) = j21I(\omega) = j21(0.151\angle57^\circ)$$

$$= (21\angle90^\circ)(0.151\angle57^\circ)$$

$$= 3.17\angle147^\circ \ V$$

In the time domain, the output voltage is given by

$$v_o(t) = 3.17\cos(3t + 147^\circ) \ V$$
Example 8:
Consider the circuit shown in Figure 15. The input to the circuit is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$.

![Figure 15](image15.png)

**Figure 15** The circuit considered in Example 6.

**Solution:** The circuit shown in Figure 15 is very similar to the circuit shown in Figure 13. There is only one difference: the dot of the left-hand coil is located at the right of the coil in Figure 13 and at the left of the coil in Figure 15. As in Example 7, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 16 shows the frequency domain representation of the circuit from Figure 15.

![Figure 16](image16.png)

**Figure 16** The circuit from Figure 15, represented in the frequency domain, using impedances and phasors.

The circuit in Figure 14 consists of a single mesh. Notice that the mesh current, $I(ω)$, enters the dotted end of the left-hand coil and the undotted end of the right-hand coil. Apply KVL to the mesh to get

$$5I(ω) + (j12I(ω) - j6I(ω)) + (-j6I(ω) + j15I(ω)) - 5.94∠140° = 0$$

$$5I(ω) + (j12 - j6 - j6 + j15)I(ω) - 5.94∠140° = 0$$

$$I(ω) = \frac{5.94∠140°}{5 + j(12 - 6 - 6 + 15)} = \frac{5.94∠140°}{5 + j15} = \frac{5.94∠140°}{15.8∠71.6} = 0.376∠68.4° \text{ A}$$
Notice that the voltage, \( V_o(\omega) \), across the right-hand coil and the mesh current, \( I(\omega) \), adhere to the passive convention. The voltage across the right-hand coil is given by

\[
V_o(\omega) = j \times 15 I(\omega) - j \times 6 I(\omega) = j \times 9 I(\omega) = j \times 9 (0.376\angle 68.4^\circ) \\
= (9\angle 90^\circ) (0.376\angle 68.4^\circ) \\
= 3.38\angle 158.4^\circ \text{ V}
\]

In the time domain, the output voltage is given by

\[
v_o(t) = 3.38 \cos(3t + 158.4^\circ) \text{ V}
\]

Example 9:
Consider the circuit shown in Figure 17. The input to the circuit is the voltage of the voltage source, \( v_s(t) \). The output is the voltage across the right-hand coil, \( v_o(t) \). Determine the steady-state output voltage, \( v_o(t) \).

**Figure 17** The circuit considered in Example 9.

**Solution:** The input voltage is a sinusoid and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input voltage. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances. Figure 18 shows the frequency domain representation of the circuit from Figure 17.

The coil currents, \( I_1(\omega) \) and \( I_2(\omega) \), and the coil voltages, \( V_1(\omega) \) and \( V_2(\omega) \), are labeled in Figure 18. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that both coil currents, \( I_1(\omega) \) and \( I_2(\omega) \), enter the undotted ends of their respective coils.
The device equations for coupled coils are:

\[
V_1(\omega) = j16 I_1(\omega) + j8 I_2(\omega) \tag{7}
\]

and

\[
V_2(\omega) = j8 I_1(\omega) + j20 I_2(\omega) \tag{8}
\]

The coils are connected in parallel, consequently \( V_1(\omega) = V_2(\omega) \). Equating the expressions for \( V_1(\omega) \) and \( V_2(\omega) \) in Equations 7 and 8 gives

\[
j16 I_1(\omega) + j8 I_2(\omega) = j8 I_1(\omega) + j20 I_2(\omega)
\]

\[
j8 I_1(\omega) = j12 I_2(\omega)
\]

\[
I_1(\omega) = \frac{3}{2} I_2(\omega)
\]

Apply Kirchhoff’s Current Law (KCL) to the top node of the coils to get

\[
I(\omega) = I_1(\omega) + I_2(\omega) = \frac{3}{2} I_2(\omega) + I_2(\omega) = \frac{5}{2} I_2(\omega)
\]

Therefore

\[
I_1(\omega) = \frac{3}{5} I(\omega) \quad \text{and} \quad I_2(\omega) = \frac{2}{5} I(\omega) \tag{9}
\]

Substituting the expressions for \( I_1(\omega) \) and \( I_2(\omega) \) from Equation 9 into Equation 7 gives

\[
Figure 18 \text{ The circuit from Figure 17, represented in the frequency domain, using impedances and phasors.}
\]
\[ V_1(\omega) = j16 \left( \frac{3}{5} I(\omega) \right) + j8 \left( \frac{2}{5} I(\omega) \right) = j \frac{16(3)+8(2)}{5} I(\omega) = j12.8 I(\omega) \] (10)

Apply KVL to the mesh consisting of the voltage source, resistor and left-hand coil to get

\[ 4I(\omega) + V_1(\omega) - 5.7\angle 158^\circ = 0 \]

Using Equation 10 gives

\[ 4I(\omega) + j12.8I(\omega) - 5.7\angle 158^\circ = 0 \]

Solving for \( I(\omega) \) gives

\[ I(\omega) = \frac{5.7\angle 158^\circ}{4 + j12.8} = \frac{5.7\angle 158^\circ}{13.41\angle 73^\circ} = 0.425\angle 85^\circ \ A \]

Now the output voltage can be calculated using Equation 10:

\[ V_o(\omega) = V_1(\omega) = j12.8 I(\omega) \]

\[ = j12.8 \left( 0.425\angle 85^\circ \right) \]

\[ = (12.8\angle 90^\circ) \left( 0.425\angle 85^\circ \right) = 5.44\angle 175^\circ \ V \]

In the time domain, the output voltage is given by

\[ v_o(t) = 5.44 \cos (4t + 175^\circ) \ V \]

**Example 10:**

Consider the circuit shown in Figure 19. The input to the circuit is the voltage of the voltage source, \( v_s(t) \). The output is the voltage across the right-hand coil, \( v_o(t) \). Determine the steady-state output voltage, \( v_o(t) \).

![Figure 19](image-url)
**Solution:** The circuit shown in Figure 19 is very similar to the circuit shown in Figure 17. There is only one difference: the dot of the right-hand coil is located at the bottom of the coil in Figure 17 and at the top of the coil in Figure 19. Figure 20 shows the frequency domain representation of the circuit from Figure 19.

![Figure 20](image)

The coil currents, \(I_1(\omega)\) and \(I_2(\omega)\), and the coil voltages, \(V_1(\omega)\) and \(V_2(\omega)\), are labeled in Figure 20. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that one of coil currents, \(I_1(\omega)\), enters the undotted end of the coil while the other coil current, \(I_2(\omega)\), enters the dotted end of the coil.

The device equations for coupled coils are:

\[
V_1(\omega) = j16 I_1(\omega) - j8 I_2(\omega) \tag{11}
\]

and

\[
V_2(\omega) = -j8 I_1(\omega) + j20 I_2(\omega) \tag{12}
\]

The coils are connected in parallel, consequently \(V_1(\omega) = V_2(\omega)\). Equating the expressions for \(V_1(\omega)\) and \(V_2(\omega)\) in Equations 11 and 12 gives

\[
j16 I_1(\omega) - j8 I_2(\omega) = -j8 I_1(\omega) + j20 I_2(\omega)
\]

\[
j24 I_1(\omega) = j28 I_2(\omega)
\]

\[
I_1(\omega) = \frac{28}{24} I_2(\omega) = \frac{7}{6} I_2(\omega)
\]

Apply Kirchhoff’s Current Law (KCL) to the top node of the coils to get
\[ I(\omega) = I_1(\omega) + I_2(\omega) = \frac{7}{6} I_2(\omega) + I_2(\omega) = \frac{13}{6} I_2(\omega) \]

Therefore
\[ I_1(\omega) = \frac{7}{13} I(\omega) \quad \text{and} \quad I_2(\omega) = \frac{6}{13} I(\omega) \quad (13) \]

Substituting the expressions for \( I_1(\omega) \) and \( I_2(\omega) \) from Equation 13 into Equation 11 gives
\[ V_1(\omega) = j16 \left( \frac{7}{13} I(\omega) \right) - j8 \left( \frac{6}{13} I(\omega) \right) = j \frac{16 \left( \frac{7}{13} \right) - 8 \left( \frac{6}{13} \right)}{13} I(\omega) = j 4.9 I(\omega) \quad (14) \]

Apply KVL to the mesh consisting of the voltage source, resistor and left-hand coil to get
\[ 4 I(\omega) + V_1(\omega) - 5.7 \angle 158^\circ = 0 \]

Using Equation 14 gives
\[ 4 I(\omega) + j 4.9 I(\omega) - 5.7 \angle 158^\circ = 0 \]

Solving for \( I(\omega) \) gives
\[ I(\omega) = \frac{5.7 \angle 158^\circ}{4 + j 4.9} = \frac{5.7 \angle 158^\circ}{6.325 \angle 51^\circ} = 0.901 \angle 107^\circ \quad \text{A} \]

Now the output voltage can be calculated using Equation 14:
\[ V_o(\omega) = V_1(\omega) = j 4.9 I(\omega) \]
\[ = j 4.9 (0.901 \angle 107^\circ) \]
\[ = (4.9 \angle 90^\circ)(0.901 \angle 107^\circ) = 4.41 \angle 197^\circ \quad \text{V} \]

In the time domain, the output voltage is given by
\[ v_o(t) = 4.41 \cos(4t + 197^\circ) \quad \text{V} \]