Example

Computer analysis of this circuit gives

\[ i_1 = 1.5 \text{ A}, \ v_2 = 6 \text{ V}, \ v_3 = 60 \text{ V} \text{ and } v_4 = 22.5 \text{ V} \]

How can we check that these values are correct?

Solution:

1. Check KCL.
Label nodes a and b:

Apply KCL at node a to get

\[ i_1 = \frac{v_2}{6} + \frac{v_2}{12} \]

Substituting the given values we get

\[ 1.5 = \frac{6}{6} + \frac{6}{12} \]

Hence KCL is satisfied at node a. Next, apply KCL at node b to get

\[ 5i_1 = \frac{v_3}{10} + \frac{v_4}{15} \]

Substituting the given values we get

\[ 5 \times 1.5 = \frac{60}{10} + \frac{22.5}{15} \Rightarrow 7.5 = 6 + 1.5 \]

Hence KCL is satisfied at node b.
2. Check KVL.

Apply KVL to the left mesh to get

\[ 4i_1 + v_2 - 12 = 0 \]

Substituting the given values we get

\[ 4(1.5) + 6 - 12 = 0 \]

Hence KVL is satisfied for the left mesh. Next, apply KVL to the right mesh to get

\[ 25\left(\frac{v_4}{15}\right) + v_4 - v_3 = 0 \]

Substituting the given values we get

\[ 25\left(\frac{22.5}{15}\right) + 22.5 - 60 = 0 \]

Hence KVL is satisfied for the right mesh.

3. Check Ohm’s law for equivalent resistances.

The parallel 6 \( \Omega \) and 12 \( \Omega \) resistors are equivalent to a single \( \frac{6 \times 12}{6 + 12} = 4 \) \( \Omega \) resistor. Apply Ohm’s law to this equivalent resistor to get

\[ v_2 = 4i_1 \]

Substituting the given values we get

\[ 6 = 4(1.5) \]

Hence Ohm’s law is satisfied. The equivalent 4 \( \Omega \) resistor is in series with the 4 \( \Omega \) in the given circuit. These series resistors are equivalent to a single 8 \( \Omega \) resistor. Apply Ohm’s law to this equivalent resistor to get
12 = 8i_1

Substituting the given values we get

12 = 8(1.5)

Hence Ohm’s law is satisfied. The series 25 \( \Omega \) and 15 \( \Omega \) resistors are equivalent to a single 40 \( \Omega \) resistor. That equivalent resistor is connected in parallel with the 10 \( \Omega \) in the given circuit. These parallel resistors are equivalent to a single \( \frac{10 \times 40}{10 + 40} = 8 \Omega \). Apply Ohm’s law to this equivalent resistor to get

\[ v_3 = 8(5i_1) = 40i_1 \]

Substituting the given values we get

60 = 40(1.5)

Hence Ohm’s law is satisfied.

4. Check voltage division.

Consider the series 25 \( \Omega \) and 15 \( \Omega \) resistors. Voltage division gives

\[ v_4 = \left( \frac{15}{15 + 25} \right) v_3 = \left( \frac{3}{8} \right) v_3 \]

Substituting the given values we get

22.5 = \left( \frac{3}{8} \right) 60

Hence voltage division is satisfied for the series 25 \( \Omega \) and 15 \( \Omega \) resistors. Next, notice that the parallel 6 \( \Omega \) and 12 \( \Omega \) resistors are equivalent to a single 4 \( \Omega \) resistor. That equivalent resistor is in series with the 4 \( \Omega \) in the given circuit. Voltage division gives

\[ v_2 = \left( \frac{4}{4+4} \right) 12 = 6 \text{ V} \]

Hence voltage division is satisfied.
5. Check conservation of power.

The power supplied by the sources must be equal to the power received by the resistors. That is

\[ 12i_1 + v_3(5i_1) = 4i_1^2 + \frac{v_2^2}{6} + \frac{v_2^2}{12} + \frac{v_3^2}{10} + 25\left(\frac{v_4}{15}\right)^2 + \frac{v_4^2}{15} \]

Substituting the given values we get

\[ 12(1.5) + 5\times1.5 = 4(1.5)^2 + \frac{6^2}{6} + \frac{6^2}{12} + \frac{60^2}{10} + 25\left(\frac{22.5}{15}\right)^2 + \frac{22.5^2}{15} \]

\[ 18 + 60(7.5) = 4(2.25) + 6 + 3 + 360 + 25(2.25) + 33.75 \]

\[ 468 = 468 \]

The power supplied by the sources is indeed equal to the power received by the resistors.