Exercises

Exercise 1. This circuit that consists of 4 circuit elements, numbered from 1 to 4. These circuit elements are connected together at the nodes, labeled a thru d. Determine the power absorbed by each circuit element.

Exercise 2. This circuit that consists of nine circuit elements, numbered from 1 to 9. These circuit elements are connected together at six nodes, labeled a thru f.

a. Determine the values of the currents $i_3$, $i_4$, $i_5$, $i_7$ and $i_8$ and of the voltages $v_1$, $v_2$, $v_6$ and $v_9$.

b. Determine the power absorbed each element.
Solutions

Solution 1:
We will need to calculate the current and voltage of each circuit element. How should we label the remaining element currents and voltages? Let’s try a couple of different ways and see what happens.

First try:

Apply KCL at nodes a and b to get:

\[ i_1 = 2 \implies i_1 = 2 \text{ A} \quad \text{and} \quad 2 = i_3 + 5 \implies i_3 = -3 \text{ A} \]

Apply KVL to the meshes to get:

\[ -v_2 + 6 - 2 = 0 \implies v_2 = 4 \text{ V} \quad \text{and} \quad -v_4 - 6 = 0 \implies v_4 = -6 \text{ V} \]

Then

\[ p_1 = -(2)(2) = -4 \text{ W}, \quad p_2 = -(2)(4) = -8 \text{ W}, \quad p_3 = -(3)(6) = -18 \text{ W} \quad \text{and} \quad p_4 = -(5)(-6) = 30 \text{ W}. \]

Second try: (The reference directions of the currents \(i_1\) and \(i_3\) and of the voltages \(v_2\) and \(v_4\) are all different from what they were in the first try.)

Apply KCL at nodes a and b to get:

\[ 0 = i_1 + 2 \implies i_1 = -2 \text{ A} \quad \text{and} \quad 2 + i_3 = 5 \implies i_3 = 3 \text{ A} \]
Apply KVL to the meshes to get:

\[ v_2 + 6 - 2 = 0 \quad \Rightarrow \quad v_2 = -4 \text{ V} \quad \text{and} \quad v_4 - 6 = 0 \quad \Rightarrow \quad v_4 = 6 \text{ V} \]

Then

\[ p_1 = (2)(-2) = -4 \text{ W}, \quad p_2 = (2)(-4) = -8 \text{ W}, \quad p_3 = -(3)(6) = -18 \text{ W} \quad \text{and} \quad p_4 = (5)(6) = 30 \text{ W}. \]

**Observations:**

1. The values of the currents \( i_1 \) and \( i_3 \) and of the voltages \( v_2 \) and \( v_4 \) in the second try are all -1 times the corresponding value in the first try. That is to be expected because the reference directions of the currents \( i_1 \) and \( i_3 \) and of the voltages \( v_2 \) and \( v_4 \) have all changed. For example, \( i_1 \) in the first try and \( i_1 \) in the second try refer to different currents.

2. The values of the power absorbed by the circuit element did not depend on the choice of reference direction.

**Solution 2.** Apply KCL at nodes a, b, c, d and e to get:

\[
\begin{align*}
2 & = 3 + i_4 \quad \Rightarrow \quad i_4 = -1 \text{ A} \\
0 & = 2 + i_5 + i_3 \\
i_1 + 3 & = 5 \quad \Rightarrow \quad i_3 = 2 \text{ A} \\
i_4 + i_7 + 4 & = 0 \quad \Rightarrow \quad -1 + i_7 + 4 = 0 \quad \Rightarrow \quad i_7 = -3 \text{ A} \\
i_5 + i_8 & = i_7 \quad \Rightarrow \quad -4 + i_8 = -3 \quad \Rightarrow \quad i_8 = 1 \text{ A}
\end{align*}
\]

Apply KVL to the meshes to get:

\[
\begin{align*}
v_1 + 2 - v_2 & = 0 \\
v_2 + 3 - 6 - 3 & = 0 \quad \Rightarrow \quad v_2 = 6 \text{ V} \\
-2 + v_6 + 3 - 3 & = 0 \quad \Rightarrow \quad v_6 = 2 \text{ V} \\
6 - 3 + v_9 & = 0 \quad \Rightarrow \quad v_9 = -3 \text{ V}
\end{align*}
\]

<table>
<thead>
<tr>
<th>element</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>current</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>-1</td>
<td>-4</td>
<td>5</td>
<td>-3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>voltage</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>power absorbed</td>
<td>12</td>
<td>-12</td>
<td>-4</td>
<td>-3</td>
<td>-12</td>
<td>10</td>
<td>18</td>
<td>3</td>
<td>-12</td>
</tr>
</tbody>
</table>