Example:
The input to this circuit is the voltage of the voltage source, $v_i(t)$. The output is the capacitor voltage, $v_o(t)$. Determine the transfer function, impulse response and step response of this circuit.

Solution:
The transfer function is

$$ H(s) = \frac{R_2}{R_2 Cs + 1} + \frac{1}{R_1 + \frac{R_2}{R_2 Cs + 1}} $$

Using $R_1 = 2 \ \Omega$, $R_2 = 8 \ \Omega$ and $C = 5 \ F$ gives

$$ H(s) = \frac{0.1}{s + 0.125} $$

The impulse response is $h(t) = \mathcal{L}^{-1} [H(s)] = 0.1 e^{-0.125t} u(t) \ \text{V}$.

The step response is

$$ \mathcal{L}^{-1} \left[ \frac{H(s)}{s} \right] = \mathcal{L}^{-1} \left[ \frac{0.1}{s(s + 0.125)} \right] = \mathcal{L}^{-1} \left[ \frac{0.8}{s} - \frac{0.8}{s + 0.125} \right] = 0.8 \left(1 - e^{-0.125t}\right) u(t) \ \text{V} $$

Example:
The input to a linear circuit is the voltage, $v_i(t)$. The output is the voltage, $v_o(t)$. The impulse response of the circuit is

$$ h(t) = 12 t e^{-4t} u(t) \ \text{V} $$

Determine the step response of this circuit.

Solution:
The transfer function is:
The Laplace transform of the step response is:

\[
H(s) = \mathcal{L}[12te^{-4t}u(t)] = \frac{12}{(s+4)^2} = \frac{12}{s^2 + 8s + 16}
\]

The Laplace transform of the step response is:

\[
H(s) = \frac{12}{s(s+4)^2} = \frac{3}{4s} + \frac{-3}{(s+4)^2} + \frac{k}{s+4}
\]

The constant \(k\) is evaluated by multiplying both sides of the last equation by \(s(s+4)^2\).

\[
12 = \frac{3}{4}(s+4)^2 - 3s + ks(s+4) = \left(\frac{3}{4} + k\right)s^2 + (3+4k)s + 12 \Rightarrow k = -\frac{3}{4}
\]

The step response is

\[
\mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \left(\frac{3}{4} - e^{-4t}\left(3t + \frac{3}{4}\right)\right)u(t) \text{ V}
\]

**Example:**

The input to this circuit is the voltage source voltage, \(v_i(t)\). The output is the voltage, \(v_o(t)\). Specify value of \(R\), \(C\) and \(L\) that cause the transfer function of this circuit to be

\[
H(s) = \frac{V_o(s)}{V_i(s)} = \frac{15 \times 10^6}{(s + 2000)(s + 5000)}
\]

**Solution:**

The transfer function can also be calculated form the circuit itself. The circuit can be represented in the frequency domain as
We can save ourselves some work be noticing that the 10000 ohm resistor, the resistor labeled \( R \) and the op amp comprise a non-inverting amplifier. Thus

\[
V_a(s) = \left(1 + \frac{R}{10000}\right)V_c(s)
\]

Now, writing node equations,

\[
\frac{V_c(s) - V_i(s)}{1000} + C_s V_c(s) = 0 \quad \text{and} \quad \frac{V_o(s) - V_a(s)}{Ls} + \frac{V_o(s)}{5000} = 0
\]

Solving these node equations gives

\[
H(s) = \frac{1}{1000C} \left(1 + \frac{R}{10000}\right) \frac{5000}{L}
\]

Comparing these two equations for the transfer function gives

\[
\left(s + \frac{1}{1000C}\right) = (s + 2000) \quad \text{or} \quad \left(s + \frac{1}{1000C}\right) = (s + 5000)
\]

\[
\left(s + \frac{5000}{L}\right) = (s + 2000) \quad \text{or} \quad \left(s + \frac{5000}{L}\right) = (s + 5000)
\]

\[
\frac{1}{1000C} \left(1 + \frac{R}{10000}\right) \frac{5000}{L} = 15 \times 10^6
\]

The solution isn’t unique, but there are only two possibilities. One of these possibilities is

\[
\left(s + \frac{1}{1000C}\right) = (s + 2000) \quad \Rightarrow \quad C = 0.5 \ \mu F
\]

\[
\left(s + \frac{5000}{L}\right) = (s + 5000) \quad \Rightarrow \quad L = 1 \ \Omega
\]

\[
\frac{1}{1000\left(0.5 \times 10^6\right)} \left(1 + \frac{R}{10000}\right) \frac{5000}{1} = 15 \times 10^6 \quad \Rightarrow \quad R = 5 \ \text{k}\Omega
\]
Example:
The input to this circuit is the voltage of the voltage source, \( v_1(t) \). The output is the capacitor voltage, \( v_o(t) \). Determine the step response of this circuit.

Solution:

\[
\begin{align*}
V_a(s) &= \left(\frac{4}{4 + 2s}\right) V_i(s) = \left(\frac{2}{s + 2}\right) V_i(s) \\
V_b(s) &= \left(\frac{12}{6 + \frac{12}{s}}\right) V_i(s) = \left(\frac{2}{s + 2}\right) V_i(s) \\
V_o(s) &= \left(\frac{12}{s + 2}\right) V_i(s) = \left(\frac{2}{s + 2}\right) V_i(s) = \left(\frac{2}{s + 2}\right) V_i(s) = \left(\frac{2}{s + 2}\right) V_i(s) = \left(\frac{2}{s + 2}\right) V_i(s)
\end{align*}
\]

The transfer function is:

\[
H(s) = \frac{V_o(s)}{V_i(s)} = \frac{20}{(s + 2)^2}
\]

The Laplace transform of the step response is:

\[
V_o(s) = \frac{20}{s(s+2)^2} = \frac{5}{s} + \frac{-5}{s+2} + \frac{-10}{(s+2)^2}
\]

Taking the inverse Laplace transform:

\[
v_o(t) = 5 - 5e^{-2t}(1 + 2t) u(t) \ V
\]
Example:

The input to this circuit is the voltage source voltage, $v_i(t)$. The output is the voltage, $v_o(t)$. Specify value of $k$, $C$ and $L$ that cause the step response of this circuit to be

$$v_o(t) = [2 + 4e^{-3t} - 6e^{-2t}]u(t) \text{ V}$$

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From the circuit:

$$H(s) = \left(\frac{4}{4 + Ls}\right)(k)\left(\frac{1}{Cs}\right) = \left(\frac{4}{L}\right)(k)\left(\frac{1}{6C}\right)$$

From the given step response:

$$H(s) = \mathcal{L}\left[(2 + 4e^{-3t} - 6e^{-2t})u(t)\right] = \frac{2}{s} + \frac{4}{s + 3} - \frac{6}{s + 2} = \frac{12}{s(s + 3)(s + 2)}$$

so

$$H(s) = \frac{12}{s(s + 3)(s + 2)}$$

Comparing the two representations of the transfer functions let $\frac{1}{6C} = 3 \Rightarrow C = \frac{1}{18} \text{ F}$, $\frac{4}{L} = 2 \Rightarrow L = 2 \text{ H}$ and $2 \times 3 \times k = 12 \Rightarrow k = 2 \text{ V/V}$.