Design Problems

**Example:**

The input of this circuit is the voltage source voltage, $v_s$. The output is the resistor voltage, $v_o$. Design this circuit to have the step response

$$v_o = 5te^{-4t}u(t) \text{ V}$$

**Solution:**

Equating the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

$$V_o(s) = \frac{kR}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{5}{(s+4)^2}$$

Equating the poles:

$$s_{1,2} = -\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}} = -4 \pm j0$$

Summarizing the results of these comparisons:

$$\frac{R}{2L} = 4, \quad R = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad \frac{kR}{L} = 5$$

Pick $L = 1 \text{ H}$, then $k = 0.625 \text{ V/V}$, $R = 8 \Omega$ and $C = 0.0625 \text{ F}$. 
Example:

The input of this circuit is the voltage source voltage, $v_s$. The output is the resistor voltage, $v_o$. Design this circuit to have the step response

$$v_o = 5e^{-4t} \sin(2t)u(t) \ V$$

Solution:

Equating the Laplace transform of the step response of the given circuit to the Laplace transform of the given step response:

$$V_o(s) = \frac{kr}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{10}{(s+4)^2 + 4} = \frac{10}{s^2 + 8s + 20}$$

Equating coefficients:

$$\frac{R}{L} = 8, \quad \frac{1}{LC} = 20 \quad \text{and} \quad \frac{kr}{L} = 10$$

Pick $L = 1 \ H$, then $k = 1.25 \ V/V$, $R = 8 \ \Omega$ and $C = 0.05 \ F$.

Example:

The input of this circuit is the voltage source voltage, $v_s$. The output is the resistor voltage, $v_o$. Design this circuit to have the step response

$$v_o = 5(e^{-2t} - e^{-4t})u(t) \ V$$
Solution:
Equating the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

\[
V_o(s) = \frac{kR}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{5}{s+2} - \frac{5}{s+4} = \frac{10}{s^2 + 6s + 8}
\]

Equating coefficients:

\[
\frac{R}{L} = 6, \quad \frac{1}{LC} = 8 \quad \text{and} \quad \frac{kR}{L} = 10
\]

Pick \( L = 1 \) H, then \( k = 1.667 \) V/V, \( R = 6 \) \( \Omega \) and \( C = 0.125 \) F.

Example:

The input of this circuit is the voltage source voltage, \( v_s \). The output is the resistor voltage, \( v_o \).

Design this circuit to have the step response

\[
v_o = 5(e^{-2t} + e^{-4t})u(t) \quad \text{V}
\]

Solution:
Comparing the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

\[
V_o(s) = \frac{kR}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \neq \frac{5}{(s+2)} + \frac{5}{(s+4)} = \frac{10s + 30}{s^2 + 6s + 8}
\]

These two functions can not be made equal by any choice of \( k, R, C \) and \( L \) because the numerators have different forms.