Chapter 8

Highlights:

1. Plastic deformation is caused by dislocation motion, involving bond breaking and reforming.

2. Dislocations of certain orientations will move more easily than others. Slip systems (plane and direction of slip) fail in a certain hierarchy, where the weakest slips first. The planes of highest planar atomic density and the directions of highest linear atomic density slip more easily. Be able to calculate and understand the resolved shear stress.

3. Slip in polycrystalline materials occurs by a slightly different mechanism than slip in single crystal materials. Slip in the individual grains occurs as in a single crystal material, but at grain boundaries the dislocation motion gets deflected, since the next grain is misoriented. This makes polycrystalline materials stronger.

4. Materials can be strengthened by hindering dislocation motion, by solid solution hardening, by grain size reduction, and by strain hardening.

5. Understand and be able to numerically calculate grain growth.

Notes:

Connections to other material:

1. Chapter 8 gives the microscopic picture of plastic deformation, introduced in chapter 7.

2. Slip is easier in crystal planes of high planar atomic density crystal planes and along crystal directions of high linear atomic density. These concepts were introduced in chapter 3.

3. Annealing reduces grain size by atomic diffusion (introduced in chapter 6).

In the 1930's theoretical calculations suggested that materials should be much more resistant to plastic deformation than they actually were, leading to the suggestion that a new entity called a dislocation could exist. This theoretical prediction was confirmed experimentally in the 1950's with the development of the electron microscope. You should review edge, screw and mixed dislocations.

Slip: Process by which plastic deformation is produced by dislocation motion.
Slip plane: The plane along which the dislocation line travels.
Dislocation moves like a caterpillar, as in figures 8.1 to 8.3.
Dislocation density = (Total dislocation length)/(Unit volume).

Dislocations are associated with strain fields arising from tensile and compressive strain in the vicinity of the dislocation. You can think of the strain field as being a vector pointing from the region of compressive strain to that of tensile strain. The strain field stores some of the energy that is expended in plastic deformation through dislocation motion. About 95% of the energy put into plastic deformation is dissipated through heat, the remaining 5% is stored in the strain fields. Show figure 8.4, which graphically illustrates the strain fields near an edge dislocation. Dislocations also have a certain polarity, meaning that compressive strain exists on one side of the dislocation, tensile strain on the other side. Thus, depending on their relative orientations, they can exert either attractive or repulsive forces on each other. Show figure 8.5.
Slip Systems

Show figure 8.9. Clearly, slip occurred on a series of parallel planes. Exactly which planes slip along which direction depends on the combination of the direction of applied stress and the inherent ease of slipping for specific planes and directions. The slip system is the combination of the plane and direction most likely to slip. These are typically the most densely packed plane (highest planar atomic density) and the most densely packed direction (highest linear atomic density). Slip along a dense packed plane is less “bumpy,” since bonds are broken and reformed more often during slip in a dense-packed plane. The most common slip systems are shown in table 8.1.

**Table 8.1** Slip Systems for Face-Centered Cubic, Body-Centered Cubic, and Hexagonal Close-Packed Metals

<table>
<thead>
<tr>
<th>Metals</th>
<th>Slip Plane</th>
<th>Slip Direction</th>
<th>Number of Slip Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu, Al, Ni, Ag, Au</td>
<td>{111}</td>
<td>{110}</td>
<td>12</td>
</tr>
<tr>
<td><strong>Face-Centered Cubic</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α-Fe, W, Mo</td>
<td>{110}</td>
<td>{211}</td>
<td>12</td>
</tr>
<tr>
<td><strong>Body-Centered Cubic</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α-Fe, W</td>
<td>{211}</td>
<td>{211}</td>
<td>12</td>
</tr>
<tr>
<td>α-Fe, K</td>
<td>{321}</td>
<td>{211}</td>
<td>24</td>
</tr>
<tr>
<td><strong>Hexagonal Close-Packed</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cd, Zn, Mg, Ti, Be</td>
<td>{0001}</td>
<td>{1120}</td>
<td>3</td>
</tr>
<tr>
<td>Ti, Mg, Zr</td>
<td>{1010}</td>
<td>{1120}</td>
<td>3</td>
</tr>
<tr>
<td>Ti, Mg</td>
<td>{1011}</td>
<td>{1120}</td>
<td>6</td>
</tr>
</tbody>
</table>
Example: FCC (110) plane, see figure 3.26.

Planar atomic density = \( \frac{\text{# of atoms}}{\text{Area of unit cell}} \)

Planar atomic density = \( \frac{(2 \text{ atoms})}{(2\sqrt{2}R)(4R)} = \frac{\sqrt{2}}{8R^2} \)

Review the concepts of a family of planes or directions. The \{100\} family of planes is denoted by \{}, and it includes the (100), (010) and (001) planes. If you draw these planes in any crystal system, they will have the same atomic configuration and are for practical purposes equivalent. The <100> family of directions is denoted by <>, and it includes the [100], [010] and [001] directions. If you draw these directions in any crystal system, they will encounter the same atomic spacing and are for practical purposes equivalent.

Show figure 8.6. In the FCC crystal system, slip occurs in the \{111\} planes and along the <110> directions.

The \{111\} family of planes includes (111), \( \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \), \( \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \) and \( \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \). Within the (111) plane shown in figure 8.6, there are 3 directions from the <110> family. Can you name them? Table 8.1 shows that this slip system contains 12 elements, equal to 4 planes x 3 directions per plane. The larger the number of slip systems, the more easily slip can occur.

Slip in Single Crystals

Show figure 8.7. Three directions are involved.
1. Direction of applied stress.
2. Direction of slip.
3. Perpendicular to slip plane.

\( \phi \)- angle between directions 1 & 3.
\( \lambda \)- angle between directions 1 & 2.

Slip involves shear stress, but a tensile stress puts shear stress on all planes not either parallel or perpendicular...
to the applied stress. What is this stress? It is obtained from purely geometrical arguments.

\[ \tau_R = \text{resolved shear stress} = \sigma \cos \phi \cos \lambda \]

When \( \phi = 0^\circ \), then \( \lambda = 90^\circ \) and \( \tau_R = 0 \).
When \( \phi = 90^\circ \), then \( \tau_R = 0 \).
Thus, the resolved shear stress is zero when the plane of interest is either parallel or perpendicular to the applied stress.
When \( \lambda = 0^\circ \), then \( \tau_R \) is maximum.

In some cases, it is useful to be able to determine the angle between two different directions, \([u_1v_1w_1]\) and \([u_2v_2w_2]\) using the following formula, equation (8.6):

\[
\cos \theta = \frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{u_1^2 + v_1^2 + w_1^2} \sqrt{u_2^2 + v_2^2 + w_2^2}}
\]

If the applied stress is gradually increased until the material deforms plastically, which plane will fail? This will occur when the shear stress reaches a critical value, the critically resolved shear stress \( \tau_{crss} \), which represents the minimum shear stress required to initiate slip. We can calculate the minimum stress that might initiate yielding in a single crystal sample, following equation (8.4) in the book.

\[
\sigma_y = \frac{\tau_{crss}}{(\cos \phi \cos \lambda)_{\text{max}}}
\]

The global minimum yield stress occurs at \( \phi = \lambda = 45^\circ \), so only under these exact conditions:

\[
\sigma_y = 2 \tau_{crss}
\]

**Example:**

A tensile stress is applied to single crystal Ag along the [001] direction. Slip occurs on a (111) plane and in the [101] direction and is initiated at an applied tensile stress of 1.1 MPa. What is \( \tau_{crss} \)?

We need to figure out the angle between the applied stress and both the slip direction and the normal to the slip plane. Remember that the direction normal to the (111) plane is the [111] direction.

What is \( \cos \phi \), the angle between the [001] and [111] directions? You can draw a right triangle with sides of length \( a, \sqrt{2}a, \sqrt{3}a \) and the angle \( \phi \) is opposite the side of length \( \sqrt{2}a \). Then,

\[
\cos \phi = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}
\]
What is $\cos \lambda$, the angle between the [001] and $\begin{pmatrix} -1 & 0 & 1 \end{pmatrix}$ directions? You can draw a right triangle with sides of length $a$, $a$, $\sqrt{2}a$ and the angle $\lambda$ is opposite one side of length $a$. Then,

$$\cos \lambda = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

Now rearrange equation (8.4):

$$\tau_{c rss} = \sigma_y \cos \phi \cos \lambda$$

$$\tau_{c rss} = 1.1 \text{ MPa} \left( \frac{1}{\sqrt{3}} \right) \left( \frac{1}{\sqrt{2}} \right) = 0.45 \text{ MPa}$$

**Polycrystalline Materials**

Show figures 8.10 and 8.11. Slips occurs along the slip system most favorably aligned within each grain. Polycrystalline materials are stronger than single crystal materials because grain boundaries constrain slip because slip planes and directions in adjacent grains are usually poorly aligned. The dislocation often must change directions when it encounters a grain boundary, and this slows slip.

**Strengthening of Materials**

1. **Grain size reduction.** Fine-grained materials are stronger, as shown from equation (8.7),

$$\sigma_y \approx \sigma_0 + k_y d^{-1/2}$$

Here $d$ is the average grain diameter. Show figure 8.15.
Example:

a) From figure 8.15, determine the constants $\sigma_0$ and $k_y$. The easiest way to do this is to choose two points from the graph, then use equation (8.7) to setup two equations and two unknowns. Choose the points (0, 25 MPa) and (13 mm$^{-1/2}$, 200 MPa).

\[
25 \text{ MPa} = \sigma_0 + k_y(0)
\]
\[
200 \text{ MPa} = \sigma_0 + k_y\left(13 \text{ mm}^{-1/2}\right)
\]

This is an easy set of equations to solve, since the 1st equation yields $\sigma_0 = 25$ MPa. Substitution into the 2nd yields $k_y = 13.5$ MPa-mm$^{1/2}$.

b) What is $\sigma_y$ when the average grain diameter is $1.0 \times 10^{-3}$ mm. This corresponds to $d^{1/2} = 31.6$ mm$^{1/2}$. Substituting,

\[
\sigma_y = 25 \text{ MPa} + \left(13.5 \text{ MPa} - \text{mm}^{1/2}\right)(31.6 \text{ mm}^{-1/2})
\]

\[
\sigma_y = 452 \text{ MPa}
\]

2. **Solid solution hardening**. Alloys are usually harder than pure metals because substitutional impurities normally impose lattice strains on the host lattice. Show figure 8.17. The additional lattice strains hinder the propagation of dislocations through the material. Show figure 8.16.

3. **Strain hardening** arises due to the interactions if strain fields. Staining a material typically pushes
the dislocations closer together, and since on average strain field interactions are repulsive, the material becomes stronger. Strain hardening can be characterized by the amount of cold work,

$$\%CW = \left( \frac{A_0 - A_d}{A_0} \right) \times 100\%$$

where $A_0$ is the initial cross-sectional area and $A_d$ that after plastic deformation. Show figure 8.19.

Example:

Circular cross section: Cold-rolled from $D = 15.2$ mm to $D = 11.4$ mm.
Rectangular cross section: Cold-rolled from $125 \times 175$ mm to $75 \times 200$ mm.

Which of these samples will be the hardest after plastic deformation and why? Strain hardening is greatest for the sample with the greatest $\%CW$ from equation (8.6).

$$\left(\%CW\right)_{circ} = 100 \left( \frac{A_0 - A_d}{A_0} \right) = 100 \left( \frac{\pi \frac{D^2}{4} - \pi \frac{D_d^2}{4}}{\pi \frac{D_0^2}{4}} \right)$$

$$\left(\%CW\right)_{circ} = 100 \left( \frac{D_0^2 - D_d^2}{D_0^2} \right) = 44\%$$

$$\left(\%CW\right)_{rect} = 100 \left( \frac{A_0 - A_d}{A_0} \right) = 100 \left( \frac{125 \times 175 \, mm^2 - 75 \times 200 \, mm^2}{125 \times 175 \, mm^2} \right)$$

$$\left(\%CW\right)_{rect} = 31\%$$

Since the $\%CW$ done on the circular sample is greater, this sample will have greater strain hardening.