PROBLEM 5.15

KNOWN: Diameter and radial temperature of AISI 1010 carbon steel shaft. Convection coefficient and temperature of furnace gases.

FIND: Time required for shaft centerline to reach a prescribed temperature.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

PROPERTIES: AISI 1010 carbon steel, Table A.1 (T = 550 K): \( \rho = 7832 \text{ kg/m}^3 \), \( k = 51.2 \text{ W/m} \cdot \text{K} \), \( c = 541 \text{ J/kg K} \), \( \alpha = 1.21 \times 10^{-5} \text{ m}^2/\text{s} \).

ANALYSIS: The Biot number is

\[
Bi = \frac{hr_o/2}{k} = \frac{100 \text{ W/m}^2 \cdot \text{K} (0.05 \text{ m/2})}{51.2 \text{ W/m} \cdot \text{K}} = 0.0488.
\]

Hence, the lumped capacitance method can be applied. From Equation 5.6,

\[
\frac{T - T_\infty}{T_1 - T_\infty} = \exp \left[ -\left( \frac{hA_s}{\rho Vc} \right) t \right] = \exp \left[ -\frac{4h}{\rho cD} t \right]
\]

\[
\ln \left( \frac{800 - 1200}{300 - 1200} \right) = \frac{-0.811}{4 \times 100 \text{ W/m}^2 \cdot \text{K}} = \frac{7832 \text{ kg/m}^3 \cdot (541 \text{ J/kg K}) \cdot 0.1 \text{ m}}{1200 - 400} = 0.436
\]

\[
t = \ln \left( \frac{0.434}{0.05} \right) = 859 \text{ s}.
\]

COMMENTS: To check the validity of the foregoing result, use the one-term approximation to the series solution. From Equation 5.52c,

\[
\frac{T_0 - T_\infty}{T_1 - T_\infty} = \frac{-400}{-900} = 0.444 = C_1 \exp \left( -\zeta_1^2 Fo \right)
\]

For Bi = hr_o/k = 0.0976, Table 5.1 yields \( \zeta_1 = 0.436 \) and \( C_1 = 1.024 \). Hence

\[
\frac{-(0.436)^2 \left( 1.2 \times 10^{-5} \text{ m}^2/\text{s} \right) t}{\left( 0.05 \text{ m} \right)^2} = \ln (0.434) = -0.835
\]

\[
t = 915 \text{ s}.
\]

The results agree to within 6%. The lumped capacitance method underestimates the actual time, since the response at the centerline lags that at any other location in the shaft.
PROBLEM 5.23

KNOWN: Thickness and properties of strip steel heated in an annealing process. Furnace operating conditions.

FIND: (a) Time required to heat the strip from 300 to 600 °C. Required furnace length for prescribed strip velocity \( V = 0.5 \text{ m/s} \), (b) Effect of wall temperature on strip speed, temperature history, and radiation coefficient.

SCHEMATIC:

ASSUMPTIONS: (1) Constant properties, (2) Negligible temperature gradients in transverse direction across strip, (c) Negligible effect of strip conduction in longitudinal direction.

PROPERTIES: Steel: \( \rho = 7900 \text{ kg/m}^3 \), \( cp = 640 \text{ J/kg-K} \), \( k = 30 \text{ W/m-K} \), \( \varepsilon = 0.7 \). 

ANALYSIS: (a) Considering a fixed (control) mass of the moving strip, its temperature variation with time may be obtained from an energy balance which equates the change in energy storage to heat transfer by convection and radiation. If the surface area associated with one side of the control mass is designated as \( A_s \), \( A_{sc} = A_{sr} = 2A_s \) and \( V = \delta A_s \) in Equation 5.15, which reduces to

\[
\rho \delta \varepsilon \sigma \left[ T^4 - T_{sur}^4 \right] = 2 \left[ h(T - T_{in}) + \varepsilon \sigma \left( T^4 - T_{sur}^4 \right) \right]
\]

or, introducing the radiation coefficient from Equations 1.8 and 1.9 and integrating,

\[
T_f - T_i = -\frac{1}{\rho \delta \varepsilon \sigma \left( \delta/2 \right)} \int T_f^t \left[ h(T - T_{in}) + h(T - T_{sur}) \right] dt
\]

Using the IHT Lumped Capacitance Model to integrate numerically with \( T_i = 573 \text{ K} \), we find that \( T_f = 873 \text{ K} \) corresponds to

\[
t_f \approx 209 \text{s}
\]

in which case, the required furnace length is

\[
L = V t_f \approx 0.5 \text{ m/s} \times 209 \text{ s} \approx 105 \text{ m}
\]

(b) For \( T_w = 1123 \text{ K} \) and 1273 K, the numerical integration yields \( t_f \approx 102 \text{s} \) and 62s respectively. Hence, for \( L = 105 \text{ m} \), \( V = L/t_f \) yields

\[
V(T_w = 1123 \text{ K}) = 1.03 \text{ m/s}
\]

\[
V(T_w = 1273 \text{ K}) = 1.69 \text{ m/s}
\]

Continued...
PROBLEM 5.23 (Cont.)

which correspond to increased process rates of 106% and 238%, respectively. Clearly, productivity can
be enhanced by increasing the furnace environmental temperature, albeit at the expense of increasing
energy utilization and operating costs.

If the annealing process extends from 25°C (298 K) to 600°C (873 K), numerical integration
yields the following results for the prescribed furnace temperatures.

As expected, the heating rate and time, respectively, increase and decrease significantly with increasing
$T_w$. Although the radiation heat transfer rate decreases with increasing time, the coefficient $h_r$ increases
with $t$ as the strip temperature approaches $T_w$.

**COMMENTS:** To check the validity of the lumped capacitance approach, we calculate the Biot number
based on a maximum cumulative coefficient of $(h + h_r) \approx 300 \text{ W/m}^2\cdot\text{K}$. It follows that $Bi = (h + h_r)(\delta/2)/k = 0.06$ and the assumption is valid.
PROBLEM 5.57

KNOWN: Initial temperature, thickness and thermal diffusivity of glass plate. Prescribed surface temperature.

FIND: (a) Time to achieve 50% reduction in midplane temperature, (b) Maximum temperature gradient at that time.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: Prescribed surface temperature is analogous to \( h \to \infty \) and \( T_{\infty} = T_S \). Hence, \( Bi = \infty \). Assume validity of one-term approximation to series solution for \( T(x,t) \).

(a) At the midplane,

\[
\theta_0^* = \frac{T_0 - T_s}{T_i - T_s} = 0.50 = C_1 \exp \left( -\zeta_1^2 Fo \right)
\]

\[
\zeta_1 \tan \zeta_1 = Bi = \infty \rightarrow \zeta_1 = \pi / 2.
\]

Hence

\[
C_1 = \frac{4 \sin \zeta_1}{2 \zeta_1 + \sin (2 \zeta_1)} = \frac{4}{\pi} = 1.273
\]

\[
Fo = -\frac{\ln \left( \theta_0^* / C_1 \right)}{\zeta_1^2} = 0.379
\]

\[
t = \frac{FoL^2}{\alpha} = \frac{0.379 (0.01 \text{ m})^2}{6 \times 10^{-7} \text{ m}^2 / \text{s}} = 63 \text{ s}.
\]

(b) With \( \theta^* = C_1 \exp \left( -\zeta_1^2 Fo \right) \cos \zeta_1 x^* \)

\[
\frac{\partial T}{\partial x} = \frac{(T_i - T_s)}{L} \frac{\partial \theta^*}{\partial x^*} = -\frac{(T_i - T_s)}{L} \zeta_1 C_1 \exp \left( -\zeta_1^2 Fo \right) \sin \zeta_1 x^*
\]

\[
\left. \frac{\partial T}{\partial x} \right|_{\text{max}} = \left. \frac{\partial T}{\partial x} \right|_{x^* = 1} = -\frac{300 \degree C}{0.01 \text{ m}} \frac{\pi}{2} 0.5 = -2.36 \times 10^4 \degree C / \text{m}.
\]

COMMENTS: Validity of one-term approximation is confirmed by \( Fo > 0.2 \).
PROBLEM 5.66

KNOWN: Long plastic rod of diameter D heated uniformly in an oven to $T_i$ and then allowed to convectively cool in ambient air ($T_\infty$, $h$) for a 3 minute period. Minimum temperature of rod should not be less than 200°C and the maximum-minimum temperature within the rod should not exceed 10°C.

FIND: Initial uniform temperature $T_i$ to which rod should be heated. Whether the 10°C internal temperature difference is exceeded.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Uniform and constant convection coefficients.

PROPERTIES: Plastic rod (given): $k = 0.3$ W/m·K, $\rho c_p = 1040$ kJ/m$^3$·K.

ANALYSIS: For the worst case condition, the rod cools for 3 minutes and its outer surface is at least 200°C in order that the subsequent pressing operation will be satisfactory. Hence,

$$
Bi = \frac{ht_0}{k} = \frac{8 \text{ W/m}^2 \cdot \text{K} \times 0.015 \text{ m}}{0.3 \text{ W/m} \cdot \text{K}} = 0.40
$$

$$
Fo = \frac{t^2}{t_0^2} = \frac{0.3 \text{ W/m} \cdot \text{K}}{1040 \times 10^3 \text{ J/m}^3 \cdot \text{K}} \times \frac{3 \times 60}{(0.015 \text{ m})^2} = 0.2308.
$$

Using Eq. 5.52a and $\zeta_1 = 0.8516$ rad and $C_1 = 1.0932$ from Table 5.1,

$$
\theta^* = \frac{T(t_0, t) - T_\infty}{T_i - T_\infty} = C_1 J_0(\zeta_1 t_0^*) \exp(-\zeta_1^2 Fo).
$$

With $t_0^* = 1$, from Table B.4, $J_0(\zeta_1 \times 1) = J_0(0.8516) = 0.8263$, giving

$$
\frac{200 - 25}{T_i - 25} = 1.0932 \times 0.8263 \exp\left(-0.8516^2 \times 0.2308\right)
$$

$T_i = 254°C$. <

At this time (3 minutes) what is the difference between the center and surface temperatures of the rod? From Eq. 5.52b,

$$
\frac{\theta^*}{\theta_o} = \frac{T(r_0, t) - T_\infty}{T(0, t) - T_\infty} = \frac{200 - 25}{T(0, t) - 25} = 0.8263
$$

which gives $T(0,t) = 237°C$. Hence,

$$
\Delta T = T(0, 180s) - T(r_0, 180s) = (237 - 200)°C = 37°C.
$$

Hence, the desired max-min temperature difference sought (10°C) is not achieved.

COMMENTS: $\Delta T$ could be reduced by decreasing the cooling rate; however, $h$ can not be made much smaller. Two solutions are (a) increase ambient air temperature and (b) non-uniformly heat rod in oven by controlling its residence time.
**PROBLEM 5.71**

**KNOWN:** A ball bearing is suddenly immersed in a molten salt bath; heat treatment to harden occurs at locations with $T > 1000$ K.

**FIND:** Time required to harden outer layer of 1 mm.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) One-dimensional radial conduction, (2) Constant properties, (3) $\text{Fo} \geq 0.2$.

**ANALYSIS:** Since any location within the ball whose temperature exceeds 1000 K will be hardened, the problem is to find the time when the location $r = 9$ mm reaches 1000 K. Then a 1 mm outer layer will be hardened. Begin by finding the Biot number.

$$\text{Bi} = \frac{h r_o}{k} = \frac{5000 \text{ W/m}^2 \cdot \text{K}(0.020 \text{ m}/2)}{50 \text{ W/m} \cdot \text{K}} = 1.00.$$  

Using the one-term approximate solution for a sphere, find

$$\text{Fo} = -\frac{1}{\zeta_1^2} \ln \left[ \frac{\theta^* / C_1}{\zeta_1 r^*} \sin \left( \zeta_1 r^* \right) \right].$$

From Table 5.1 with $\text{Bi} = 1.00$, for the sphere find $\zeta_1 = 1.5708$ rad and $C_1 = 1.2732$. With $r^* = r/r_o = (9 \text{ mm}/10 \text{ mm}) = 0.9$, substitute numerical values.

$$\text{Fo} = \frac{-1}{(1.5708)^2} \ln \left[ \frac{(1000 - 1300) \text{ K}}{(300 - 1300) \text{ K}} / 1.2732 \frac{1}{1.5708 \times 0.9} \sin \left( 1.5708 \times 0.9 \text{ rad} \right) \right] = 0.441.$$  

From the definition of the Fourier number with $\alpha = k/\rho c$,

$$t = \text{Fo}^2 \frac{\rho c}{\alpha} = \text{Fo} \cdot r_o^2 \frac{\rho c}{k} = 0.441 \times \left[ \frac{0.020 \text{ m}}{2} \right]^2 \frac{7800 \text{ kg/m}^3 \times 500 \text{ J/kg} \cdot \text{K}}{50 \text{ W/m} \cdot \text{K}} = 3.4 \text{ s}.$$

**COMMENTS:** (1) Note the very short time required to harden the ball. At this time it can be easily shown the center temperature is $T(0, 3.4 \text{ s}) = 871$ K.
PROBLEM 5.85

KNOWN: Asphalt pavement, initially at 50°C, is suddenly exposed to a rainstorm reducing the surface temperature to 20°C.

FIND: Total amount of energy removed (J/m²) from the pavement for a 30 minute period.

SCHEMATIC:

ASSUMPTIONS: (1) Asphalt pavement can be treated as a semi-infinite solid, (2) Effect of rainstorm is to suddenly reduce the surface temperature to 20°C and is maintained at that level for the period of interest.

PROPERTIES: Table A-3, Asphalt (300K): \( \rho = 2115 \text{ kg/m}^3, \ c = 920 \text{ J/kg} \cdot \text{K}, \ k = 0.062 \text{ W/m} \cdot \text{K} \).

ANALYSIS: This solution corresponds to Case 1, Figure 5.7, and the surface heat flux is given by Eq. 5.61 as

\[
q_s^* (t) = k (T_s - T_i) / (\pi \alpha t)^{1/2}
\]

The energy into the pavement over a period of time is the integral of the surface heat flux expressed as

\[
Q^* = \int_0^t q_s^* (t) \, dt.
\]

Note that \( q_s^* (t) \) is into the solid and, hence, \( Q^* \) represents energy into the solid. Substituting Eq. (1) for \( q_s^* (t) \) into Eq. (2) and integrating find

\[
Q^* = k (T_s - T_i) / (\pi \alpha)^{1/2} \int_0^t t^{-1/2} \, dt = \frac{k (T_s - T_i)}{(\pi \alpha)^{1/2}} \times 2 \, t^{1/2}.
\]

Substituting numerical values into Eq. (3) with

\[
\alpha = \frac{k}{\rho \ c} = \frac{0.062 \text{ W/m} \cdot \text{K}}{2115 \text{ kg/m}^3 \times 920 \text{ J/kg} \cdot \text{K}} = 3.18 \times 10^{-8} \text{ m}^2 / \text{s}
\]

find that for the 30 minute period,

\[
Q^* = \frac{0.062 \text{ W/m} \cdot \text{K} \times (20 - 50) \text{K}}{(\pi \times 3.18 \times 10^{-8} \text{ m}^2 / \text{s})^{1/2}} \times 2 (30 \times 60 \text{s})^{1/2} = -4.99 \times 10^5 \text{ J/m}^2.
\]

COMMENTS: Note that the sign for \( Q^* \) is negative implying that energy is removed from the solid.