**PROBLEM 11.1**

**KNOWN:** Initial overall heat transfer coefficient of a fire-tube boiler. Fouling factors following one year’s application.

**FIND:** Whether cleaning should be scheduled.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Negligible tube wall conduction resistance, (2) Negligible changes in \( h_c \) and \( h_h \).

**ANALYSIS:** From Equation 11.1b, the overall heat transfer coefficient after one year is

\[
\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R_{f,i}^{+} + R_{f,o}^{-}.
\]

Since the first two terms on the right-hand side correspond to the reciprocal of the initial overall coefficient,

\[
\frac{1}{U} = \frac{1}{400 \text{ W/m}^2 \cdot \text{K}} + (0.0015 + 0.0005) \text{ m}^2 \cdot \text{K} / \text{W} = 0.0045 \text{ m}^2 \cdot \text{K} / \text{W}
\]

\[
U = 222 \text{ W/m}^2 \cdot \text{K}.
\]

**COMMENTS:** Periodic cleaning of the tube inner surfaces is essential to maintaining efficient fire-tube boiler operations.
**PROBLEM 11.6**

**KNOWN:** Condenser arrangement of tube with six longitudinal fins \((k = 200 \text{ W/m} \cdot \text{K})\). Condensing refrigerant at temperature 45°C flows axially through inner tube while water flows at 0.012 kg/s and 15°C through the six channels formed by the splines.

**FIND:** Heat removal rate per unit length of the exchanger.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) No heat loss/gain to the surroundings, (2) Water is incompressible liquid with negligible viscous dissipation, (3) Negligible thermal resistance on condensing refrigerant side, \(h_i \to \infty\), (4) Water flow is fully developed, (5) Negligible thermal contact between splines and inner tube, (6) Heat transfer from outer tube negligible.

**PROPERTIES:** Table A-6, Water (\(T_c = 15°C = 288 \text{ K}\)): \(\rho = 1000 \text{ kg/m}^3\), \(k = 0.595 \text{ W/m} \cdot \text{K}\), \(\nu = \mu/\rho = 1138 \times 10^{-6} \text{ N} \cdot \text{s/m}^2/1000 \text{ kg/m}^3 = 1.138 \times 10^{-6} \text{ m}^2/\text{s}\), \(\text{Pr} = 8.06\); Tube fins (given): \(k = 200 \text{ W/m} \cdot \text{K}\).

**ANALYSIS:** Following the discussion of Section 11.2,

\[
q' = UA' \left(T_h - T_c\right)
\]

\[
\frac{1}{UA'} = R'_h + R'_w + R'_c = R'_w + \frac{1}{(\eta_0 hA')_c}
\]

where \(R'_h = 0\), due to the negligible thermal resistance on the refrigerant side (h), and

\[
R'_w = \frac{\ln(D_2/D_1)}{2\pi k} = \frac{\ln(14/10)}{2\pi (200 \text{ W/m} \cdot \text{K})} = 2.678 \times 10^{-4} \text{ m} \cdot \text{K} / \text{W}.
\]

To estimate the thermal resistance on the water side (c), first evaluate the convection coefficient. The hydraulic diameter for a passage, where \(A_c\) is the cross-sectional area of the passage is

\[
D_{h,c} = \frac{4A_c}{P} = \frac{4\left[\pi \left(D_3^2 - D_2^2\right)/4 - 6(D_3 - D_2)t/2\right]/6}{(\pi D_2 - 6t)/6 + (\pi D_3 - 6t)/6 + 2(D_3 - D_2)/2}
\]

\[
D_{h,c} = \frac{4\left[\pi (50^2 - 14^2)/4 - 6(50 - 14)\right] \times 10^{-6} \text{ m}^2/6}{[(14\pi - 6 \times 2)/6 + (50\pi - 6 \times 2)/6 + (50 - 14)] \times 10^{-3} \text{ m}}
\]

\[
D_{h,c} = \frac{4 \times 2.656 \times 10^{-4} \text{ m}^2}{6.551 \times 10^{-2} \text{ m}} = 0.01622 \text{ m}.
\]

Hence the Reynolds number is

Continued …
PROBLEM 11.6 (Cont.)

\[ \text{Re}_{D,c} = \frac{(0.012 \text{ kg/s} / 6) / \left(1000 \text{ kg/m}^3 \times 2.656 \times 10^{-4} \text{ m}^2 \right)}{1.138 \times 10^{-6} \text{ m}^2 / \text{s}} = 107 \]

and assuming the flow is fully developed,

\[ \text{Nu}_{D,c} = \frac{h_c D_{h.c}}{k} = 3.66 \]

\[ h_c = 3.66 \times 0.595 \text{ W/m} \cdot \text{K} / 0.01622 = 134 \text{ W/m}^2 \cdot \text{K}. \]

The temperature effectiveness of the splines (fins) on the cold side is

\[ \eta_o = 1 - \frac{A_{f,c}}{A_c} \left(1 - \eta_f \right) \]

where \( A_{f,c} \) and \( A_c \) are, respectively, the finned and total (fin plus prime) surface areas, while

\[ \eta_f = \frac{\tanh (mL)}{mL} \]

\[ m = (2h_c / kt)^{1/2} = \left[ \left(2 \times 134 \text{ W/m}^2 \cdot \text{K} \right) / (200 \text{ W/m} \cdot \text{K} \times 0.002 \text{m}) \right]^{1/2} = 25.88 \text{ m}^{-1} \]

\[ \eta_f = \frac{\tanh \left( 25.88 \text{ m}^{-1} \times 0.018 \text{m} \right)}{25.88 \text{ m}^{-1} \times 0.018 \text{m}} = \frac{0.4348}{0.4658} = 0.934. \]

Hence

\[ \eta_o = 1 - \frac{6(D_3 - D_2)}{6(D_3 - D_2) + (\pi D_2 - 6t)} \left[1 - \eta_f \right] \]

\[ \eta_o = 1 - \frac{6(50-14)}{6(50-14) + (14\pi - 6\times 2)} (1 - 0.934) = 0.943 \]

\[ \frac{1}{\eta_{0} \tau A_c} = \frac{1}{0.943 \times 134 \text{ W/m}^2 \cdot \text{K} \times \left[6(50-14) + (14\pi - 6\times 2)\right] \times 10^{-3} \text{ m} = 3.22 \times 10^{-2} \text{ m} \cdot \text{K} / \text{W} \]

and the heat rate is

\[ q' = \frac{T_h - T_c}{R' + 1/(\eta_{0} \tau A_c)} \]

\[ q' = \frac{(45-15) \text{ K}}{2.678 \times 10^{-4} \text{ m} \cdot \text{K} / \text{W} + 3.22 \times 10^{-2} \text{ m} \cdot \text{K} / \text{W}} = 924 \text{ W/m.} \]

COMMENTS: (1) The effective length of the fin representing the splines was conservatively estimated. The heat transfer by conduction through the splines to the outer tube and then by convection to the water was ignored.

(2) Without the splines, find \( D_h = (D_3 - D_2) = 36 \text{ mm} \) so that \( h_c = 60.5 \text{ W/m}^2 \cdot \text{K}. \) The heat rate with \( A_c' = \pi D_2 \) is

\[ q' = (hA_c') (T_h - T_c) = 60.5 \text{ W/m}^2 \cdot \text{K} (0.014\pi \text{ m}) (45-15) \text{ K} = 79 \text{ W/m.} \]

The splines enhance the heat transfer rate by a factor of \( 924/79 = 11.7. \)
**PROBLEM 11.8**

**KNOWN:** Diameter and inner and outer convection coefficients of a condenser tube. Thickness, outer diameter, and pitch of aluminum fins.

**FIND:** (a) Overall heat transfer coefficient without fins, (b) Effect of fin thickness and pitch on overall heat transfer coefficient with fins.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Negligible tube wall conduction resistance, (2) Negligible fouling and fin contact resistance, (3) One-dimensional conduction in fin.

**PROPERTIES:** Table A.1, Aluminum (T = 300 K): \( k = 237 \text{ W/m} \cdot \text{K} \).

**ANALYSIS:** (a) With no fins, Eq. 11.1 yields

\[
U = \left( \frac{1}{h_i} + \frac{1}{h_o} \right)^{-1} = \left( 2 \times 10^{-4} + 0.01 \right)^{-1} = 98.0 \text{ W/m}^2 \cdot \text{K} <
\]

(b) With fins and a unit tube length, Eqs. 11.1 and 11.3 yield

\[
U_i = \frac{1}{\pi D_i} + \frac{1}{\pi h_i D_i + \eta_o h_o A_o'}
\]

and \( \eta_o = 1 - \left( \frac{A'_f}{A'_o} \right) (1 - \eta_f) \). The total fin surface area per unit length is \( A'_f = N'2\pi \left( \frac{r_{oc}^2 - r_t^2}{r_{oc}} \right) \), where the number of fins per unit length is \( N' = 1m/S(m) \). The total outside surface area per unit length is \( A'_o = A'_f + (1 - N'\pi)D_i \), and the fin efficiency is given by Eq. 3.96 or Fig. 3.20.

For \( t = 0.0015 \text{ m} \) and \( S = 0.0035 \text{ m} \), \( r_{oc} = (D_o/2) + (t/2) = 0.01075 \text{ m} \), \( N' \approx 286 \), \( A'_f = 0.163 \text{ m}^2/\text{m} \), and \( A'_o = (0.163 + 0.018) \text{ m}^2/\text{m} = 0.181 \text{ m}^2/\text{m} \). With \( r_{oc}/r_t = 2.15 \), \( L_c = 0.00575 \text{ m} \), \( A_p = 8.625 \times 10^{-6} \text{ m}^2 \), and \( L_c^{3/2} \left( h_o/kA_p \right)^{1/2} = 0.0964 \), Fig. 3.20 yields \( \eta_f \approx 0.99 \). Hence, \( \eta_o \approx 1 - (0.163/0.181)(0.01) = 0.99 \) and

\[
U_i = \left[ \frac{1}{\pi D_i} + \frac{1}{\eta_o h_o A_o'} \right]^{-1}
\]

\[
U_i = \left[ \frac{2 \times 10^{-4} \text{ m}^2 \cdot \text{K}/\text{W} + \pi \times 0.01 \text{m}/0.99 \times 100 \text{W}/\text{m}^2 \cdot \text{K} \times 0.181 \text{ m}^2/\text{m} }{L_c^{3/2} \left( h_o/kA_p \right)^{1/2}} \right]^{-1} = 512 \text{ W/m}^2 \cdot \text{K} <
\]

We may use the IHT Extended Surface Model (Performance Calculations for a Circular Rectangular Fin Array) to consider the effect of varying \( t \) and \( S \). To maximize \( N' \), the minimum allowable value of

Continued...
S - t = 1.5 mm should be selected. It is then a matter of choosing between a large number of thin fins or a smaller number of thicker fins. Calculations were performed for the following options.

<table>
<thead>
<tr>
<th>t (mm)</th>
<th>S (mm)</th>
<th>N’</th>
<th>U_i (W/m²·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>400</td>
<td>640</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>286</td>
<td>512</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>222</td>
<td>460</td>
</tr>
<tr>
<td>4</td>
<td>5.5</td>
<td>182</td>
<td>420</td>
</tr>
</tbody>
</table>

Since heat transfer increases with $U_i$, the best configuration corresponds to $t = 1$ mm and $S = 2.5$ mm, which provides the largest airside surface area.

**COMMENTS:** The best performance is always associated with a large number of closely spaced fins, so long as the flow between adjoining fins is sufficient to maintain the convection coefficient.
PROBLEM 11.26

KNOWN: Flowrates and inlet temperatures of a cross-flow heat exchanger with both fluids unmixed. Total surface area and overall heat transfer coefficient for clean surfaces. Fouling resistance associated with extended operation.

FIND: (a) Fluid outlet temperatures, (b) Effect of fouling, (c) Effect of UA on air outlet temperature.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Constant properties, (3) Negligible tube wall resistance.

PROPERTIES: Air and gas (given): \( c_p = 1040 \text{ J/kg} \cdot \text{K} \).

ANALYSIS: (a) With \( C_{min} = C_h = 1 \text{ kg/s} \times 1040 \text{ J/kg} \cdot \text{K} = 1040 \text{ W/K} \) and \( C_{max} = C_c = 5 \text{ kg/s} \times 1040 \text{ J/kg} \cdot \text{K} = 5200 \text{ W/K} \), \( C_{min}/C_{max} = 0.2 \). Hence, \( \text{NTU} = UA/C_{min} = 35 \text{ W/m}^2 \cdot \text{K}(25 \text{ m}^2)/1040 \text{ W/K} = 0.841 \) and Fig. 11.14 yields \( \varepsilon \approx 0.57 \). With \( C_{min}(T_{h,i} - T_{c,i}) = 1040 \text{ W/K}(500 \text{ K}) = 520,000 \text{ W} = q_{max} \), Eqs. (11.20) and (11.21) yield

\[
\begin{align*}
T_{h,o} &= T_{h,i} - \varepsilon q_{max}/C_h = 800 \text{ K} - 0.57(520,000 \text{ W})/1040 \text{ W/K} = 515 \text{ K} < \\
T_{c,o} &= T_{c,i} + \varepsilon q_{max}/C_c = 300 \text{ K} + 0.57(520,000 \text{ W})/5200 \text{ W/K} = 357 \text{ K} < 
\end{align*}
\]

(b) With fouling, the overall heat transfer coefficient is reduced to

\[
U_f = \left( U^{-1} + R_f^{-1} \right)^{-1} = \left[ (0.029 + 0.004) \text{ m}^2 \cdot \text{K/W} \right]^{-1} = 30.7 \text{ W/m}^2 \cdot \text{K}
\]

This 12% reduction in performance is large enough to justify cleaning of the tubes.

(c) Using the Heat Exchangers option from the IHT Toolpad to explore the effect of UA, we obtain the following result.

\[
\begin{array}{c}
\text{Heat transfer parameter, } UA/\text{W/K} \\
\text{Air outlet temperature, } T_{co}/\text{K}
\end{array}
\]

The heat rate, and hence the air outlet temperature, increases with increasing UA, with \( T_{co} \) approaching a maximum outlet temperature of 400 K as \( UA \to \infty \) and \( \varepsilon \to 1 \).

COMMENTS: Note that, for conditions of part (a), Eq. 11.32 yields a value of \( \varepsilon = 0.538 \), which reveals the level of approximation associated with reading \( \varepsilon \) from Fig. 11.14.
**PROBLEM 11.57**

**KNOWN:** Dimensions of counterflow, concentric tube heat exchanger for recovering heat from shower drains. Inlet temperatures of hot and cold water streams. Heat transfer coefficient of inner (hot) flow. Mass flow rate of outer (cold) flow.

**FIND:** (a) Heat transfer rate and outlet temperature of cold flow, (b) Heat transfer rate and outlet temperature of cold flow when helical spring provides specified outer heat transfer coefficient, (c) Daily savings if 15,000 students each take a 10-minute shower per day and cost of heating water is $0.07/kW-h.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Constant properties and steady-state conditions, (2) Negligible heat transfer to surroundings, (3) Fully developed flow in the annular gap, (4) Uniform surface temperature correlation is appropriate, (5) Inner tube wall thermal resistance is negligible.

**PROPERTIES:** Table A.6, water ($T \approx 285$ K): $k = 0.591$ W/m·K, $c_p = 4189$ J/kg·K, $\mu = 1225 \times 10^{-6}$ N·s/m², $Pr = 8.81$.

**ANALYSIS:** (a) We begin by finding the heat transfer coefficient for the flow in the annular gap. The Reynolds number is

$$Re_D = \frac{\mu m D_h}{\mu} = \frac{m D_h}{\mu} \frac{4m}{\mu A_c} = \frac{4 \times 10 \text{ kg/min}/60 \text{ min/s}}{1225 \times 10^{-6} \text{ N·s/m}^2 \times \pi (0.05 \text{ m} + 0.07 \text{ m})} = 1444$$

Thus the flow is laminar, and from Table 8.2 with $D_i/D_o = 0.71$, $Nu_i = 5.36$. Hence,

$$h_c = \frac{Nu_i k}{D_h} = \frac{5.36 \times 0.591 \text{ W/m·K}}{0.02 \text{ m}} = 158 \text{ W/m}^2 \cdot \text{K}$$

Continued...
Then the overall heat transfer coefficient is

\[
U = \left[\frac{1}{h_c} + \frac{1}{h_h}\right]^{-1} = \left[\frac{1}{158 \text{ W/m}^2 \cdot \text{K}} + \frac{1}{1/10,000 \text{ W/m}^2 \cdot \text{K}}\right]^{-1} = 156 \text{ W/m}^2 \cdot \text{K}
\]

and using the \(\varepsilon\)-NTU method, with \(C_{\min} = C_{\max} = \frac{\text{ṁ}c_p}{\text{ṁ}c_p} = 698 \text{ W/K}, C_i = 1\), we have

\[
NTU = \frac{UA}{C_{\min}} = \frac{U \pi D_L}{C_{\min}} = 156 \text{ W/m}^2 \cdot \text{K} \times \pi \times 0.05 \text{ m} \times 1 \text{ m/698 W/K} = 0.035
\]

And from Eq. 11.29a, \(\varepsilon = \frac{NTU}{(1 + NTU)} = 0.034\). Thus from Eqs. 11.18 and 11.19,

\[
q = \varepsilon C_{\min} (T_{h,i} - T_{c,i}) = 0.034 \times 698 \text{ W/K} (38 - 10)\text{°C} = 661 \text{ W}
\]

and from Eq. 11.7b,

\[
T_{c,o} = T_{c,i} + \frac{q}{C_c} = 10\text{°C} + 661 \text{ W}/698 \text{ W/K} = 11.0\text{°C}
\]

(b) The value of \(U\) changes to \(U = \left[1/9050 \text{ W/m}^2 \cdot \text{K} + 1/10,000 \text{ W/m}^2 \cdot \text{K}\right]^{-1} = 4751 \text{ W/K}\). Then

\[
NTU = 1.07, \varepsilon = 0.517, \text{ and}
\]

\[
q = \varepsilon C_{\min} (T_{h,i} - T_{c,i}) = 0.517 \times 697 \text{ W/K} (38 - 10)\text{°C} = 10,100 \text{ W}
\]

\[
T_{c,o} = T_{c,i} + \frac{q}{C_c} = 10\text{°C} + 10,100 \text{ W}/698 \text{ W/K} = 24.5\text{°C}
\]

(c) The savings is the cost of the energy transferred from the wastewater to the cold water,

\[
\text{Savings} = 10.1 \text{ kW} \times 600 \text{ s} \times 15,000/3600 \text{ s/h} \times \$0.07/\text{kW-h} = \$1767
\]

**COMMENTS:**

(1) Commercially-available devices that are used in high density buildings such as dormitories are typically installed on larger drains that collect shower water from multiple showers, rather than on individual showers. The devices use heat transfer enhancement techniques to ensure large values of the cold side heat transfer coefficient. (2) With \(x_{ft,1} = 0.05\text{Re}_D\text{Pr}_D = 13 \text{ m}\), the flow in the annular gap is not fully developed, and the actual heat transfer coefficient would be higher than predicted in part (a). (3) In part (a), the mean temperature of the cold stream is 283.5 K. Evaluation of properties at 285 K is appropriate.
**PROBLEM 11.67**

**KNOWN:** Flow rates and inlet temperatures of exhaust gases and combustion air used in a cross-flow (one fluid mixed) heat exchanger. Overall heat transfer coefficient. Desired air outlet temperature.

**FIND:** Required heat exchanger surface area.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat loss to surroundings, (3) Constant properties, (4) Gas properties are those of air.

**PROPERTIES:** Table A-4, Air ($\bar{T}_m \approx 700 \text{ K}, 1 \text{ atm}$): $c_p = 1075 \text{ J/kg} \cdot \text{K}$.

**ANALYSIS:** Using the $\epsilon$ – NTU method,

$$c_p = m_c = 10 \text{ kg/s} \times 1075 \text{ J/kg} \cdot \text{K} = 10,750 \text{ W/K}$$

$$h = m_h c_p = 15 \text{ kg/s} \times 1075 \text{ J/kg} \cdot \text{K} = 16,125 \text{ W/K}$$

Thus $C_r = C_{min}/C_{max} = 0.667$, $\epsilon = q/q_{max} = (T_{c,o} - T_{c,i})/(T_{h,i} - T_{c,i}) = 0.688$

From Eq. 11.34b,

$$NTU = -\frac{1}{C_r} \ln[C_r \ln(1-\epsilon) + 1] = -\frac{1}{0.667} \ln[0.667 \ln(1-0.688) + 1] = 2.24$$

Therefore,

$$A = NTU \times C_{min}/U = (2.24 \times 10,750 \text{ W/K})/(100 \text{ W/m}^2 \cdot \text{K}) = 241 \text{ m}^2$$

<